APPENDIX F

A PIE CHART SHOWS PROPORTION OF PUPILS WHO USE VARIOUS SOURCES TO LEARN ALGEBRAIC EQUATIONS.
APPENDIX E

QUESTIONNAIRE FOR PUPILS

The researcher is a Masters of Science education student at Bindura University of Science Education. He wishes to establish the errors and misconceptions made by form 2 pupils when solving algebraic equations. For the study to be successful you are requested to answer each item on this questionnaire as honestly and frankly as you possibly can. The information that you going to provide will be private and confidential, so I'm kindly asking for your assistance and cooperation. Data collected using this questionnaire will be used for research purposes only.

Answer all the questions by ticking in the appropriate box

1. Do you use a textbook only in the learning of algebraic equations?
   - Yes [ ]
   - No [ ]

2. Do you have access to internet sources only when you are learning algebraic equations?
   - Yes [ ]
   - No [ ]

3. Are you accessible to both internet and textbooks when studying algebraic equations?
   - Yes [ ]
   - No [ ]
APPENDIX D

AN EVALUATION OF A DETAILED LESSON PLAN FOR THE ALGEBRAIC TEST 2

EVALUATION

The lesson was conducted according to plan. Almost all pupils managed to solve equations such as \( x^2 + 3x + 2 = 0 \) correctly. Very few pupils solved correctly quadratic equation with non-zero terms to the right side of the equation such as \( x^2 + 9x + 20 = 17 \). Some pupils solved such an equation while considering the step \((x - 5)(x - 4) = 17\). Use of factorization was ineffectively handled by most pupils. For example, solution to the equation \( x^2 = 7x \), was given as \( x \cdot x = 7 \cdot x \), thus \( x = 7 \). However the lesson’s objectives were achieved to a lesser extent.

STRENGTHS

All pupils managed to write the given test. Use of question and answer during the introduction phase, helped most pupils to effectively engage in group work discussion. Feedback given helped most pupils to achieve lesson and test objectives.

WEAKNESSES

Most pupils were failing to use factorization correctly in solving algebraic equations. Some pupils failed to complete work in time.

SUGGESTIONS

The teacher gave remedial work to pupils who were failing to use factorization correctly. Also, the teacher advised and encouraged pupils to complete given work in time.

BINDURA UNIVERSITY OF SCIENCE EDUCATION

The undersigned certify that they have read and recommended to Bindura University of Science Education for acceptance research project entitled: “an analysis of errors made by zjc learners and their misconceptions in solving algebraic equations, a case of zaka west, masvingo”.

Submitted by Chihesi Onias in partial fulfillment of the requirements of the degree of Masters of Science Education – Mathematics.

………………………………………………
SUPERVISOR 1
………………………………………………
SUPERVISOR 2
………………………………………………
PROGRAMME COORDINATOR
………………………………………………
EXTERNAL EXAMINER
………………………………………………
EXTERNAL EXAMINER

DATE ………………………..
DATE …………………
SUGGESTIONS

Teacher to undergo in class remediation with pupils to ensure a 100% pass in the same test next time. The teacher to encourage pupils to use suggested textbooks and internet sources for correct and accurate manipulation of algebraic linear equations.

DECLARATION

I, … Chihesi Onias……….., declare that this research project “AN ANALYSIS OF ERRORS MADE BY ZJC LEARNERS AND THEIR MISCONCEPTION IN SOLVING ALGEBRAIC EQUATIONS, A CASE OF ZAKA WEST, MASVINGO”, is my own work except as indicated in the references and acknowledgements. It is submitted in partial fulfilment of the requirements for the degree of Masters of Science Education at Bindura University Of Science Education, Bindura, Zimbabwe. It has not been submitted before for any degree or examination in this or any other university.

Supervisor.

I ………………………………………. Declare that I have supervised this thesis and satisfied that it can be submitted to the Faculty of Science Education of Bindura University of Science Education.

Date …………………………………

Signature ……………………………..
APPENDIX C

AN EVALUATION OF A DETAILED LESSON PLAN FOR AN ALGEBRAIC TEST 1

EVALUATION

Most pupils managed to give correct solutions to most of the given questions. Question 1(a) (b) (c) and (d) was correctly attempted by most pupils. 65% of the pupils failed question 2 because the computed -7+1 giving 6 as the solution. Some handled 6x – 20 as x= 3.2. From the given test, a 100% pass was not realized due to pupils’ failure to logically solve question 3 (d). Some solved the equation 5x + x + 2 – 3x + 12 whilst considering the step 6x +2-2 = 3x + 12. A handful of pupils computed the last stage of an equation’s solution given by 4x-1as x = 1-4. However an aggregate percentage pass was obtained. The lesson was however effective conducted to ensure an attainment of the above evaluated objectives.

STRENGTHS

All pupils managed to write the test timeously. Lesson’s introductory activities were accompanied by correctly worked examples whose questions were on flash cards. Polished floors, orderly arranged classroom furniture and monitoring of pupils’ progress contributed to the conduciveness of the learning environment. Effective analysis of the test results was achieved due to class monitoring of pupils’ progress by the teacher.

WEAKNESSES

Examples discussed during the introduction phase were slightly similar to the examination questions. Above all almost all pupils made a number of misconceptions and errors during their attempt to solve algebraic linear equations.
ACKNOWLEDGEMENT

For this study to be a success a lot of people provided the researcher with valiant effort ranging from professional, emotional, social and financial support. The researcher feels greatly indebted to them and wishes to acknowledge immense value and support rendered. My sincere gratitude is expressed to Mr Mapuwei T, my project Supervisor whose resolute focus and inspirational guidance made this project a resounding success. This great unforgettable contribution were also strengthened by the contribution of my colleagues Kudubva Phillimon, Mabatsira Loveness, Benyu Nyaradzo, Jan Watson, Zhou Silvester and Magonziwa Talent of Bindura University Science Education. The researcher also extended his appreciation of the co-operation and assistance received from all the respondents who sacrifice their time and effort in answering questions. My profound appreciation also goes to Mr Munerei F, the Headmaster of Musenyereki A secondary school, for the moral and material support rendered towards the production of this study. In addition to that, I deeply appreciate the co-operation and generosity of Mrs Mverecha Loice who bore my entire financial burden and encouraged me in the study.

APPENDIX B

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This is an investigation into the errors made by pupils when solving a system of algebraic equations at Z.J.C level. The researcher used probabilistic, systematic random sampling procedures to choose a sample size of a quarter of the total targeted population. Data was collected from a sample of 30 Form 2 pupils at a secondary school. Two tests were written by the pupils and analyzed with reference to the recent literature. A research was carried out on a sample of 30 pupils. The researcher discovered that a total of nine error types appeared in the analysis of the research study. Some of these errors were transposing, switching addends, factorisation, deletion, inverse, expansion and division. Transposing and switching addends errors were classified as structural error and mechanism were suggested for their commission both from the literature and from the experiences of the researcher. An interesting finding of this study is that transposing errors occur due to oversimplification of the transposing process. Also the switch addends error appeared more frequently in algebraic than in arithmetical equation and the resulting differing. Success rates confirm the finding of the literature on the difference in difficulty between these two types of equations. The research findings also highlight the importance of subordinate skills such as division, especially among young pupils. However the study concluded that several errors and omissions were made by pupils when solving algebraic equations. The study recommended that the two principles of equality that is symmetry and transitivity need to be stressed very explicitly in the early stage of teaching algebra. Thus pupils should recognise an equal sign as a symbol relating equal quantities on two sides of an equation. Some teachers have limited content knowledge on certain sub concepts underlying solutions of algebraic equations. Participation in in-service workshops may be of great help. In such gatherings, teachers get insight knowledge on the subject content they teach.
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APPENDIX A

A DUBLICATE OF A PROGRESS RECORD BOOK WITH PUPILS MARKS IN ALGEBRAIC EQUATIONS TEST 1 AND 2.

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ZIMSEC MATHEMATICS SYLLUBUS, (2012-2013). Ministry of primary and secondary Education.


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CHAPTER 1

INTRODUCTION

1.0 INTRODUCTION

This chapter shall discuss background of the study, statement of the problem on which it also presented research objectives, questions, assumptions and significance of the study. Limitations and delimitation of the study were indicated to ensure constraints likely to be encountered in carrying out and boundaries that demarcates the study, respectively. Definitions of terms shall be given to ensure a clear understanding of the research terms.

1.1 BACKGROUND OF THE STUDY

The historical origin of the problem was based on observation, experience and critical reading of literature, whose combination resulted in the identification of the research problems. Thus several mathematics researchers and educators studied algebraic solving ability which was viewed from different approaches such as generalization, modelling and functional. Ldris (2006) sited that, the nature of algebraic solving ability inherited in each approach is sufficient to generate powerful algebraic solving problems in the classroom. The implication, thus, teachers and mathematics educators should focus on the possible problems faced by the

5.3.2 Theoretical framework

Theoretical framework help teachers to analyze the textbooks which are relevantly and currently interpret mathematical curriculum. Teachers’ ability to analyze and interpret textbooks and multimedia sources used by pupils to learn algebra, help them to create an environment which is conducive to discussion and discovery learning. Through this errors and misconceptions are corrected instantly during discussions. Through these classroom discussions, pupils feel more comfortable and confident to justify their responses, sometimes citing some references. It was noted that, “when asking pupils to elaborate an idea helps to establish norms of civility. And respect rather than criticism and doubt (Davis 2013 p35). It is recommended that from the framework pupils will be characterized by the ability to do algebraic equations, ability to do algebraic problems, ability to communicate mathematically and the ability to reason logically.

5.4 RESEARCH CONSTRAINTS

Even though there is much a close relationship between distribution of performance, statistical tests and algebraic solving ability, no maximum attention was given to the aspects of graphs. The research design was not a true experimental and the research lacked the control.
Zimbabwe Junior Certificate (ZJC) pupils as they interact with various approaches when solving algebraic equations.

Zimbabwe School Examination Council (ZIMSEC) MATHEMATICS SYLLABUS (4008/4028) for 2012-2017 was revised whose curriculum in schools encompasses a primary aim, “to enable pupils to develop their ability in mathematical problem solving.” This aim depends on five factors, specifically, skills, concepts, processes, metacognition and attitudes. To successfully solve various algebraic equations, a pupil has to apply concepts, skills, processes and metacognition. Mandler (1989) emphasizes the importance for analysing the problems with learners to ensure that errors and mistakes can be handled by some alternative routes, substitutes, actions or a rewording of the equation.

ZIMSEC REPORT (2010), for mathematics examination, suggests that the general performance of candidates in solving algebraic equations remained poor. Report for 10 November 2010 paper 2 stresses that question 2b, 4 and 7 involved algebra, whose solutions were incorrectly presented by most candidates as they continue to use incorrect formulae and failure to carry out calculations logically. ZIMSEC syllabus 4008/4028 for 2012-2017, section 3.8 and 3.9 point out that, pupils will be assessed in their ability to give steps or information necessary to solve problems and use appropriate formulae, algorithms and strategies to solve a variety of problems. However, beyond ZIMSEC’s expectations and evaluations, the goal is to investigate ways to promote performance on algebraic equations by seeking means for analysing pupils’ reaction to various representations of equations in different context while considering the way the concept shall be taught.

53 RECOMMENDATIONS

The following recommendations were made based on the findings of the study.

5.3.1 Pupils’ errors and misconceptions

1. Transition from arithmetic to algebra needs to be made as smooth as possible so that pupils can build up the arithmetic-to-algebra connections. Pupils could be helped by teachers to develop a good sense on number operations. This would help them to apply operations meaningfully and flexibly.

2. The equal sign. Two principles of equality i.e. symmetry and transitivity need to be stressed very explicitly in the early stage of teaching of algebra. Thus pupils should recognize that an equal sign as a symbol relating equal quantities on two sides of an equation.

3. Pupils should be helped to develop a better understanding of the dual nature of algebraic equations as entities.

4. Structure of an equation. In solving algebraic equations pupils need to understand why certain transformations have to be done. This helps them to remember algorithms to apply in solving either linear or quadratic equations.

5. Some teachers have limited content knowledge on certain sub-concepts underlying solution of linear equations participation in in-service workshops may be of great help. In such gatherings, teachers get insight knowledge on the subject content they teach.
5.2 CONCLUSION

The main purpose of the study was to investigate and make a critical analysis of the problems that Z.J.C pupils face when solving algebraic equations and suggest possible solutions. The study revealed that problems pupils face when solving algebraic equations are due to their errors and misconceptions they made when solving algebraic equations.

5.2.1 Pupils' conceptual and procedural knowledge

This refers to the background knowledge and skills that are required to minimize errors and misconceptions pupils make when solving algebraic equations. These comprise:

- Basic operations on directed numbers and fractions including the rules governing the order of operations and the use of brackets;
- Problem-solving skills and strategies;
- The meaning of equation and use of equal signs;
- Proper interpretation of algebraic expressions and equations;
- Algebraic problem solving strategies and designs.

The meaning of equation and use of equal signs;

Proper interpretation of algebraic expressions and equations;

Problem-solving skills and strategies.

A theoretical framework promotes an effective acquisition of mathematical knowledge and skills required to solve algebraic equations in a problem-based environment. A theoretical framework includes:

- The meaning of equation and use of equal signs;
- Proper interpretation of algebraic expressions and equations;
- Problem-solving skills and strategies.

1.2 STATEMENT OF THE PROBLEM

Algebraic linear equations are one of the most abstract strands in mathematics. Once largely limited to the secondary school curriculum, it is now common place in college mathematics courses and is often the first algebraic concepts and skills (Green and Rubenstein 2008) many are discontinuing their study of higher level mathematics because of their lack of success in algebra. The demand of algebra at more levels increases. Pupils' conceptual and procedural knowledge is essential in understanding the problems they face when solving algebraic equations.
poor performance in algebraic equations. Literature reviewed suggested that Z.J.C pupils’ logic does not follow the rules of deductive or inductive thinking. Some findings showed that pupils’ performance is haphazard. Errors in algebraic solving procedures provided valuable insights into pupils’ thinking. ‘Other inverse’ and ‘transposing’, was errors found in the reviewed literature. Some algebraic solving problems found were related to confusion and misconceptions. Differences in ‘Arithmetic’ and ‘algebraic’ equations, ‘structural’ and ‘procedural’ methods in solving equations, motivated the carrying out of the research project.

In chapter 3, the researcher formulated research methodology. Quasi experimental design was used to gather credible data. Data was drawn from a sample of 30 Form two pupils at Musenyereki Secondary school, whose marks in test were analyzed and interpreted to give the research results. The chosen sample responded to the questionnaires. The runs test, Chi-squared test, input modeling and questionnaires were used as research instruments. Effectiveness of these mathematical tests, statistical tools and computer packages were based on the research findings. Methods for data presentation analysis and interpretation were established. The methodology to this study helped the researcher to come up with research results.

In chapter 4, the runs test, Chi-squared test and questionnaires were used to come up with research results. Data collected from the research study was tabulated. These tables were presented as answers to the research questions. The runs test was used to determine randomness in the distribution of pupils’ performance in classroom. Chi-squared test was used to provide evidence on the distribution of pupils’ errors form a selected sample. Evaluation summaries of algebra is used in companies to figure out their annual budget which involve their annual expenditures. Various stores use algebra to predict the demand for particular product and subsequently place their order. Algebra also has individual applications in the form of calculation of annual taxable income, bank interests and instalments loans. Algebraic expressions and equations serve as model for interpreting and making inferences about data. Further algebraic reasoning and symbolic notation also serve as the basis for the design and use of computer spreadsheet models. Therefore mathematical reasoning developed through algebra is necessary all through life, affecting decision we make such as personal finance, travel, cooking and real estate to name a few.

Thus it can be argued that a better understanding of algebra improves decision making in societies. Teachers in the field of mathematics often have differences of opinion about learners’ conceptions and misconception. This is not only because the amount of quantitative reasoning that experts use is greater than what learners use in problem solving situation. Frequently teachers do not realize that this quality is essential to disseminate to their students. Students should be allowed to use this information which is not in the textbooks. For the teacher this knowledge is structured in their heads as informal, imagist metaphoric and heuristic forms (kaput, 1985). The problem is that this knowledge is not properly presented in the modern curricular. If happens, learners will be beneficiaries.

Although there are many causes of learner difficulties in mathematics, the lack of support from research fields for teaching and learning is noticeable. If research could characterize learners’ errors and misconceptions, it would be possible to design effective instructions to avoid situations. Research on learners’ errors and misconception is a way to provide such support for both teachers and students. Problem of this nature are particularly worthy investigation as there
is still a lack of robust research in identifying learners’ misconception for more than one conceptual area collectively. The existing research is mostly about identifying and explaining causes for a particular misconception. If researchers can identify students’ difficulties collectively in more than one area, it will be easier to identify the systematic patterns of errors that spread through the areas and make suggestions and remediations.

Another point is that there is a methodological shift in modern research for classical studies in mathematics education which were statistical statement about population to a closer observation of individuals doing mathematics tasks. In this context, this study is significant because it addresses errors made by ZJC learners and their misconceptions in algebraic solving tasks. I hope that addressing this will issue reduce the distance between research and the real classroom leading to more practically applicable finding.

1.3 OBJECTIVES OF THE STUDY

The purpose of the research is to investigate the problems that ZJC pupils encounter when solving algebraic equations. The aim is to be achieved through the following objectives;

- To assess pupils’ errors and misconceptions they face in solving algebraic equations.
- To assess possible sources of pupils’ incorrect solutions to given algebraic equations.
- To identify the relevant background knowledge, concepts and sub concepts for the understanding of algebraic equations.
- To identify suitable methods to use for the problems identified.

CHAPTER 5
SUMMARY, CONCLUSIONS AND RECOMMENDATIONS

5.0 INTRODUCTION

This chapter summarizes the research findings. Briefings from all the previous chapters were given. Conclusions based on the research findings addressed the research problem and objectives. Special recommendations were outlined basing with the research results. Constraints encountered in this research study were also given in this chapter.

5.1 SUMMARY

The main purpose of this research was to identify and critically analyze the problems that ZJC pupils face when solving algebraic equations. From the background of research problem the researcher came up with chapter 1. In this research study, answers to the five research questions were obtained which serve as solutions to research objectives. The research questions were, is pupils’ performance within a classroom random, what are common errors pupils make when solving algebraic equations, what are the possible sources of pupils’ incorrect solutions to given algebraic equations, what theoretical framework could be used for analyzing pupils’ reasoning power about algebraic equations and what are the promising methods for learning of algebraic equations? The related literature helped the researcher to come up with some solutions to the identified research questions.

In chapter 2 findings in current ZIMSEC mathematics syllabus showed that pupils are required to develop ability in mathematical problem solving. Some ZIMSEC’s mathematics reports proved a
1.4 RESEARCH QUESTIONS

In an attempt to find the answers to the research problems, the study provides answers to the following sub-problems:

1. What are pupils’ categories of error and misconception in solving algebraic equations.
2. What are possible sources of pupils’ incorrect solutions to given algebraic equations.
3. What are the relevant background knowledge, concepts and sub concepts required for the understanding of algebraic equations.
4. What are the promising methods for the problem identified.

1.5 ASSUMPTIONS

The study shall be guided by the following assumptions;

- Teachers are aware of various participatory methodologies and are not hesitant to employ them during teaching and learning of algebraic equations.
- Pupils and teachers will be willing to take part in the research process.
- Pupils’ responses to the questionnaires will positively expose true and sincere attitudes towards the use of participatory methods to give the study a high degree of validity and reliability.
- The sample of pupils chosen will represent the total population at the school hence they will reflect their characteristics and expresses their views and solutions when learning algebraic equations.

4.6.2 Step 2

Remembering the rules of equation solving is the most essential topic of an algebraic course. Pupils were advised to simplify one side of an equation and apply the same operation to both sides of an equation. Simplifying one side of an equation means expanding products, factoring polynomials, placing fractions over a common denominator and simplifying a numerical expression. Applying the same operation on both sides refers to the effective use of multiplicative and additive inverses. However, from the above steps an acronym PEMDAS was used where P is the parenthesis, E is the exponent, M is multiplication, D is the division, A is the addition and S is the subtraction.

4.7 SUMMARY

The findings revealed addressed the research problem. From this chapter, the runs test proved that randomly chosen pupils at Musenyereki A Secondary school have random algebraic errors. The chi-squared test used data summary from input modelling to address some of the research questions. Questionnaires were effectively used to give credible data concerning sources used by pupils when solving algebraic equations at ZJC level. A theoretical framework could have improved understanding and presentation of algebraic solving abilities.
1.6 SIGNIFICANCE OF THE STUDY

The purpose of this study is to identify problems likely to be faced by pupils in their attempt to solve algebraic equations. Therefore, the study may benefit the learners, teachers and also the researcher.

1.6.1 Learners

Equations are a central part of any mathematics course, especially at Z.J.C level. Pupils may benefit since it encourages their participation in learning situations, thus leading them to appreciate algebraic equations as an integral part of a wide variety of algebraic, geometric and trigonometric problems. Pupils may also be able to apply algebraic problem solving techniques in other sciences and social sciences such as physics and economics respectively. Pupils may have a rich opportunity to apply their mathematical knowledge in physical situations, in real world and use algebra as a problem solving tool.

1.6.2 Teachers

The analysis of the possible mechanisms responsible for problems faced by pupils when solving algebraic equations may facilitate the design and development of improved teaching strategies in the classroom. Novice teachers may be able to improve the quality of their teaching in algebra not only by being aware of errors or misconceptions but also by alerting pupils to such errors while the initial teaching is taking place. The study could also help the teacher to develop effective pupils’ initial thinking processes during the learning of equations.

1.6.3 The researcher

As a teacher to be, the researcher may have benefited from the research also. The study might improve the competence of the researcher by providing appropriate methods and skills which have been exposed and originated from social classes where access to textbooks is limited. These pupils rely on chalkboard work whose reasoning ability is not concrete level. The implication affects algebraic content knowledge and algebraic pedagogical knowledge. The impact also affects logical reasoning skills in written work. Teachers should analyze and evaluate pupils’ reasoning power that is deductive and inductive logical abilities. From the general algebraic solving techniques used by a pupil, a teacher can identify his/her social background and sources he/she used. Teachers and mathematical educators should plan, implement and possess relevant algebraic pedagogical content knowledge to enhance algebra learning style. At Musenyereki A Secondary most pupils were from rural areas. These pupils had a low probability of being exposed to internet sources and their reasoning was concrete such a group of pupils did not require lectured method in their learning of algebraic equations. Pupils from surrounding communities performed if exposed to library hardcopies provided an effective use of media under discovery learning.

4.6 THE PROMising METHODS IN THE LEARNING OF ALGEBRAIC EQUATIONS.

There is a credibility that randomly chosen pupils pass or fail algebraic equations tests. Pupils who passed or failed algebraic equations made algebraic equations misconceptions and error, regardless of the teaching/learning methods and sources used. However the following algebraic problem solving techniques could be an appropriate method in solving equations. This method could ensure even distribution of performance.

4.6.1 Step 1

Begin by knowing how to do multiplication, division, addition, subtraction handle exponents, ratios and fractions. Teachers should help pupils to think of \( a^2 + b^2 = c^2 \), \( d = rt \) and \( y = mx + b \) when approaching new and unfamiliar equations.
enhance effective teaching of algebraic equations at Z.J.C. Through the study, the researcher could have been aware of the actual problems pupils face when solving algebraic equations.

1.7 LIMITATIONS OF THE STUDY
There were drawbacks the researcher faced during the research process. The time taken by the researcher was relatively short (6 months) so the results may not be 100% true, but if it was a longitudinal project, the result may be more perfect. Secondly due to financial constraints the researcher could not raise enough stationary like the duplicating papers to set questions and questionnaires for pupils and teachers to cater for the whole population. Otherwise a large sample (not 30 participants) could have been considered therefore it comprise the result from being 100% true. Apart from that, transport problem due to shortage of fuel as well as to high transport cost hindered the researcher to visit as many schools as possible and hence the study was carried out at only one school. And worse to that, not all pupils in form 2 are involved because only a small sample is to be chosen. Therefore results are limited as generalization or true reflection of what will happen to all Z.J.C pupils in Zimbabwe. In addition to the study, the research had tough time visiting some schools which were not accessible. The study was conducted during normal working hours. Each time permission had to be sought from the school head and the heads of schools being reached on. Some pupils may give poor solution due to fear of the examination, which therefore distort the research findings. Also, if the researcher structure the instructions poorly, it means pupils may find it difficult to interpret the questions. As a result the research’s outcome can be ineffective.

1.8 DELIMITATIONS
The research was to be conducted at Musenyereki A Secondary School, in Masvingo province where the researcher was teaching in the year 2017. Research subjects involve selected pupils in

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4.5 THEORETICAL FRAMEWORK USED FOR ANALYZING PUPILS REASONING ABOUT ALGEBRAIC EQUATIONS?
The conceptual framework could help to analyze reasoning abilities in algebraic equations.

Fig 1. shows a theoretical framework used to solve and analyze solving abilities for an algebraic equation in any randomly chosen pupil.
form 2 classes. The study was based on Z.J.C pupils at Musenyereki A secondary school only. The selected pupils represent the Z.J.C pupils at Musenyereki A Secondary.

1.8.0 DEFINITION OF TERMS

1.8.1 Conception and Misconception
Learner beliefs, their theories, meanings and explanation will form the basis of the term learner conception. When those conceptions are deemed to be in conflict with respected meaning in Mathematics, then a misconception has occurred, (Osbornne and Wittrock 1983). While recognizing the substantial theoretical diversity of meaning I define two over changing terms in my study- errors and misconceptions that many of the above theoretical underpinnings.

1.8.2 Error
In Mathematics an error means the deviation from a correct solution of a problem, in this study an error is regarded as a mistake in the process of solving a mathematical problem algorithmically, procedurally or by any other method. Error could be found in wrongly answered problem which have flaws in the process that generated the answers. (Young and Osheba, 1981)

1.8.3 Algebra
Filloy and Rojan (2003) define algebra as a part of Mathematics in which letters and other general symbols are used to present numbers and quantities in formulae and equations. From the above definition algebra can be defined as a branch of Mathematics marked which is chiefly by the use of symbols to present numbers. Thus a branch of Mathematics that substitute letters for numbers. Algebra is a branch of mathematics that uses mathematical statements to describe relationship between things that vary over time.

4.4 POSSIBLE SOURCES OF PUPILS’ INCORRECT SOLUTIONS TO THE GIVEN ALGEBRAIC EQUATIONS?

Table 4.4.0 Proportions of pupils who use textbooks and internet source when studying algebraic equations

<table>
<thead>
<tr>
<th>Sources used by pupils</th>
<th>Number of pupils</th>
<th>Percentage of pupils</th>
</tr>
</thead>
<tbody>
<tr>
<td>Textbooks only</td>
<td>31</td>
<td>62</td>
</tr>
<tr>
<td>Internet sources only</td>
<td>6</td>
<td>12</td>
</tr>
<tr>
<td>Both textbooks and internet</td>
<td>13</td>
<td>26</td>
</tr>
</tbody>
</table>

Table 4.4.0 above highlighted the proportion of pupils who use variety of sources when learning algebraic equations. A pie chart in appendix C shows these proportions greatest proportion used textbooks. This result was related with the findings of Hall (2012) who found how language and learning procedures in textbooks cause problems in the learning of algebraic equations. The smallest proportion from the pie chart used internet sources. However, some algebraic solving errors were caused by use of internet sources as proposed by Hypes and Sutherland (2005).

Findings from Lewis (2000), Channon et al (1992) and Alick et al (2010) showed that there are different approaches used when introducing how to solve equations involving “difference of two squares.” internet sources use various mathematical symbols of division and multiplication. This yielded errors and misconceptions, Z.J.C pupils faced in solving algebraic equations. However all pupils, who both passed and failed algebraic tests faced challenges in an attempt to comprehend and interpret abstract algebraic language, notations and symbols. The pupils eventually failed to solve new and unfamiliar equations.
Decision rule:

\[ H_0 : \chi^2_{\text{cal}} > \chi^2_{0.05} = 11.10 \]

Accept \( H_0 ^{1.8.4 Equation} \) if \( \chi^2_{\text{cal}} > \chi^2_{0.05} = 11.10 \)

From the input modelling results, the chi-squared calculated value is 4.08. However, \( 4.08 < 11.10 \). The conclusion follows that we fail to reject \( H_0 \) and errors made by pupils follow a probability distribution. There was likelihood that every pupil made errors when solving algebraic equations. Both the fail and the pass group had misconceptions in solving equations. These errors are not consistent because they occur at random. However errors and misconceptions found from the research study have been grouped with respect to linear equations and quadratic equations.

Error 1 under linear equations was discovered by Hall (2012). Error 2 was identified as the omission error. Demonstration by Martz (2009) showed that the deduction-reduction process of solving an equation, increased confusion in the presentation of work. This omission error was caused by the complexity of the question. Differences in the “Arithmetic” and “Algebraic” equations found by Orton (2011) were a source of pupils' misconception for the omission error.

Error 3 under linear equations was found in the literature by Kieran (1992). This error type was referred to as the transposing error. Errors in quadratic equation were related to the Innate High Cognitive Demand of algebra found by Kieran (1992). Findings from Hoyles and Sutherland (2005) on language, symbols and learning procedures used in textbooks, influenced learning of algebraic equations. Factorization errors were related to the procedural and structural nature of an algebraic equation.

1.8.4 Equation

Hall (2012) defines an equation as written statement indicating the equality of two expressions which consist of a sequence of symbols that is split into left and right sides joined by an equal sign. From the above definition an equation can be referred as a statement that the value of two mathematical expressions are equal (indicated by the sign).

1.8.5 Algebraic expression and equation

Hall (2012) states that an algebraic expression is an expression which is built up from integers constants, variables and the algebraic operations (addition subtraction, multiplication and division and exponentiation by an exponent that is a ration number). An equation is a mathematical statement of equality between algebraic expressions.

That is an equation is algebraic if it involves a finite combination of numbers, variables and algebraic operations i.e. addition, subtraction, multiplication, division, raising to a power and extracting a root.

1.8.6 Solving algebraic equation

Davis 2013) states that solving an algebraic equation is done by adding, subtracting, multiplying and dividing both sides of the equation by numbers and variables, so that one would end up with a single variable on one side and a single number on the other side. It implies isolating the variable meaning, to get the letter on one side of the equation.

1.8.7 Problem

Carry (2007) define a problem as a perceived gap between the existing state and a desired state or a deviation from a norm. Thus any question or matter involving doubt,
uncertainty or difficulty. A matter or situation regarded as unwelcome or harmful and needing to be dealt with and overcome.

➢ An inquiry starting from given conditions to investigating or demonstrate a fact result or law.

1.8.8 Summary
Chapter 1 defines the problem and sub-divide into sub-problems, objectives, assumptions importance, limitations of the study were highlighted. Also key terms were defined to show that the research has direction and meaning. The next chapter reviewed the related literature which helps the researcher to be aware of some findings under research and found related solutions to the research problem.

4.3 ERRORS MADE BY PUPILS WHEN SOLVING ALGEBRAIC EQUATIONS

Table 4.3.0 shows errors made by pupils in algebraic test 1 and 2

<table>
<thead>
<tr>
<th>Error</th>
<th>Nature of Test</th>
<th>Factorisation Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1. Number line error</td>
<td>$-7 + 1 \neq 6$</td>
</tr>
<tr>
<td>2</td>
<td>2. Inverse error</td>
<td>$4x = 1$</td>
</tr>
<tr>
<td>3</td>
<td></td>
<td>$x = 1 - 4$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Error</th>
<th>Algebraic equations</th>
<th>Algebraic quadratic equations</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$x^2 - 9x + 20 = 17$</td>
<td>$(x - 5)(x - 4) = 17$</td>
</tr>
<tr>
<td>2</td>
<td>$x^2 = 7x$</td>
<td>$x = 7$</td>
</tr>
<tr>
<td>3</td>
<td>$x(x - 1) = 12$</td>
<td></td>
</tr>
</tbody>
</table>

Table 4.3.0 above is errors pupils made in solving linear algebraic equations. The results were obtained from appendices D and E. To determine whether these errors are consistent, the researcher used chi-squared test as goodness-of-fit-test. He tested the following hypothesis;

$H_0 :$ pupils' errors follow a probability distribution

$H_1 :$ pupils' errors do not follow a probability distribution.
CHAPTER 2
LITERATURE REVIEW

2.0 INTRODUCTION
This chapter discusses what has been said by other researchers on the challenges and difficulties, Z.J.C pupils face when solving algebraic equations. Information researched from books, internet, journals and magazines were discussed in this chapter. The researcher examines some of the available literature highlighting some of the issues relating to the possible solutions to the difficulties faced by pupils.

2.1 CRITICAL REVIEW
There is an extensive literature on error analysis in algebraic equations at Z.J.C level. Hebert and Carpenter (2009) summed that, one of the potential implication of research on students’ errors is that, instruction might be designed to address directly the specific deficits that help to diagnosis errors and misconceptions. Bell -Grader (2013), points out that, childhood logic is transductive, meaning that it does not follow the rules of deductive or inductive thinking for any given algebraic equation. Greeno (2003) found that pupils' performance appeared to be quite haphazard. Generally any randomly chosen pupil at Z.J.C has specific algebraic solving problems inherited from randomly used sources such as textbooks and multimedia sources.

\[
\frac{28 - 25.46}{\sqrt{12.205}}
\]

= 0.716

Significance level:

\[
\frac{0.05}{2}
\]

= 0.025

Decision rule:

Reject \( H_0 \) if \( z_0 > 0.025 = 1.96 \)

Accept \( H_1 \) if \( z_0 < 1.96 \)

Since \( z_0 = 0.716 < 1.96 \), the researcher accepted \( H_0 \) and a conclusion was drawn. However from the conclusion, the distribution of marks obtained by pupils was random.

This means, pupils' performance in algebraic equations at Z.J.C levels varies from pupil to pupil. This also proved finding from Greeno (2003) and Carry (2007) who suggests that pupils' performance in algebraic equations appear to be quite haphazard. Generally any randomly chosen pupil at Z.J.C has specific algebraic solving problems inherited from randomly used sources such as textbooks and multimedia sources.
forced to confront explicitly the conflict between their misconceptions and scientific principles, the two may coexist as separate islands of knowledge.

Literature concerning "procedural" and "structural" approaches views of nature of algebraic equations. Some features of linear equations might not be appreciated by novice teachers. Kieran (1992), suggest that solutions to linear equations are of a procedural nature. This implies that arithmetic operations are carried out on numbers to yield numbers. For example, procedural approach was found in the solution of the equation;

\[ 2x + 1 = 5 \]

Where various numerical values of \( x \) might be tried until a solution is found. As Kieran (1992) notes, the objects that are operated on are not the algebraic expression but their numerical instantiations. This stage of pupils' mathematical education is the introduction of formal algebra.

According to Kieran (1992), this stage of formal algebra, teacher instruction should move away from procedural nature towards a structural approach. Structural approach refers to a different set of operations that are carried out, not on numbers but on algebraic expressions. For example, the solution to:

\[ 3x + 4 = 9 \]

Can involve the step;

\[ 3x + 4 - 4 = 9 - 4 \]

Which has nothing to do with either final solution or any numerical instantiations? Libnowicz (2008) suggest that structural approach requires a grasp of the structure of an equation. As Kieran, (1992, p392), states, “the implicit objectives of school algebra are structural, then an

(2003) who found that pupils’ performance appeared to be haphazard. The following hypothesis was used;

\[ H_0: \text{the marks are random} \]
\[ H_1: \text{the marks are not random} \]

It follows that, Mean of the marks = \[ \frac{\sum n}{N} \] 27 (using calculator in statistical mode)

The pluses and the minuses of the data are as follows

+ + + − − − + + + + + − − − − − − − + + + + + + + + + − − − − − − − − − − + − + − − − + − + − + − + − − + − + − + − − − − − − − − − − − =

\[ n_1 = 24 \]
\[ n_2 = 26 \]
\[ b = 28 \]

where \( n_1 \) is the number of “+”, \( n_2 \) is the number of “−”, \( b \) is the number of runs

\[ \mu_b = \frac{2n_1n_2}{N} + \frac{1}{2} \]
\[ = \frac{2(24)(26)}{50} + \frac{1}{2} \]
\[ = 25.46 \]

\[ \delta^2_b = \frac{2n_1n_2(Nn_1n_2 - N)}{N^2(N - 1)} \]
\[ = \frac{2(24)(26)[2(24)(26) - 50]}{50^2[50 - 1]} \]
\[ = 12,205 \]

Test statistic:

\[ z_0 = \frac{b - [\frac{2n_1n_2}{N} + \frac{1}{2}]}{\delta} \]
after summing up marks in test 1 and 2. Appendix B is a list of the data obtained from column headed “total marks” in appendix A. Appendix C and D are the evaluation of detailed lesson plans for algebraic equations tests 1 and 2 written by the pupils and recorded. Appendix E is a questionnaire designed to identify pupils who use variety of sources in the learning of algebraic expressions. Appendix F is a pie chart used to compare proportions of pupils who use textbooks and multimedia sources. However data from these appendices were used to answer research questions.

4.2 DISTRIBUTION OF PUPIL’S PERFORMANCE IN ALGEBRAIC EQUATIONS

Table 4.2.0: Results for algebraic test 1 and 2

<table>
<thead>
<tr>
<th>Nature of test</th>
<th>Number of pupils passed</th>
<th>Number of pupils failed</th>
<th>Total number of pupils</th>
</tr>
</thead>
<tbody>
<tr>
<td>Algebraic linear equations (test 1)</td>
<td>09</td>
<td>21</td>
<td>30</td>
</tr>
<tr>
<td>Algebraic quadratic equations (test 2)</td>
<td>19</td>
<td>11</td>
<td>30</td>
</tr>
<tr>
<td>Total</td>
<td>29</td>
<td>21</td>
<td>60</td>
</tr>
</tbody>
</table>

Results on Table 4.2.0 above shows that the total number of pupils who passed and failed algebraic equations. Data used was indicated in Appendix A and B. To determine the distribution of pupils’ performance in algebraic equations, the researcher tested for the randomness of the data given in appendix B. This was done to prove findings from Greeno effective teacher should have a clear understanding of the shift from the procedural towards structural approaches.

Language and learning procedures in textbooks are a source of pupils’ misconceptions and problems in solving algebraic equations. Channon, Smith and Macrae (1992) lay out fig 22.3 which demonstrate a discovery approach in the learning of “differences of two squares.” Alick, Apollo, Dzvuke and Nyamakura (2010) introduce equations involving “difference of two squares” using indices under lecture method demonstrated on an example. Lewis (2000) uses a box method under deductive/inductive approach to introduce the same concept. However, Hall (2012) points that textbooks and multimedia are the sources of pupil’s problems in algebraic equations. Hoyles and Sutherland (2005) concludes that language, symbols and learning approaches used in textbooks and on internet sources influenced modern day learning of algebraic equations.

The literature also views differences in the “arithmetic” equation and “algebraic” equations. Orton (2011) refers to an “arithmetical” equation having the form:

\[ ax + b = c \]

And “algebraic” equation having the form:

\[ ax + b = cx. \]

Filloy and Romano (2003) classify these linear equations whose analysis is in the light of their theories. Having recognized “algebraic” rather than “arithmetical” type of equation, Orton (2011) suggest that pupils face further problems because of the dual nature of the equal signs. Martz (2009) views syntactic similarity between semantically different statements. For instance;
CHAPTER 4

DATA PRESENTATION, ANALYSIS AND DISCUSSION.

4.0 INTRODUCTION

The researcher would present, analyze and interpret the research data collected at Musenyereki ‘A’ Secondary school, that is the in the Z.J.C classes. He tested for the randomness of the marks obtained by the pupils using the runs test. The runs test is non-parametric statistical test that checks a randomness hypothesis for the data sequence. In this research, the runs test was used to examine whether pupils’ performance in algebraic equations is occurring randomly from pupil to pupil. It also determined whether pupils’ errors in algebraic equations occur in sequence or over time. Chi-squared is a non-parametric data used as goodness-of-fit test to determine whether pupils’ errors and misconceptions in algebraic equations are consistent. Chi-squared test compares observed data values with expected data values, drawn from input modeling. Questionnaires were used to derive data which was presented on a pie chart. The pie chart should help to analyze proportion of pupils’ who used textbooks and multimedia sources in the learning of algebraic equation equations. A theoretical framework was drawn to suggest possible solutions to the problems faced by pupils when solving algebraic equations.

4.1 RESEARCH RESULTS

Appendices A, B, C, D, E and F are the possible sources for the research data analysis and interpretation. Appendix A contains a record of marks in algebraic equations test 1 and 2 written by Form 2 pupils. Sample of transcripts showing some errors made by pupils are also attached after appendix A. Column headed “total marks” in appendix A give the research data, obtained

\[
4x + 12 = 4(x + 3)
\]

And

\[
3x + 3 = 2x + 7
\]

Were viewed as a serious obstacle in algebra. Hall (2012), suggests that an equation;

\[
4x + 12 = 4(x + 3)
\]

Is a tautology because the left hand side and the right hand side are syntactically equivalent whereas the second equation;

\[
3x + 3 = 2x + 7
\]

Is not tautology and requires a new procedure to solution of the equation. Martz (2009) described a major structural confusion which can arise in solving linear equations. Maverech and Vitschak (2013) concluded that, from the pupils they tested had a poor understanding of the meaning of the equal signs. Hence an explicit way may help to give many pupils a more structural understanding of linear equations.

Skeman (2013), suggests that confusion is going from arithmetic to algebra. For example, Martz (2009) argues that; \(3 \frac{3}{4}\) and \(3 + \frac{3}{4}\); Is not unreasonable that the student should interpret algebraic expression: \(3x\) as \(3 + x\).

Therefore, there is plenty of room for confusion and misinterpretation in the initial stage of algebraic equations. Problems in solving algebraic equations are related to some errors pupils
Procedure 7

By the help of the above test statistics and the decision rule a conclusion of the test was made. That stated whether to accept the null hypothesis or reject it.

However, conclusion drawn from the input modelling, runs tests and chi-squared test were coordinated systematically with the evaluation summaries from the algebraic test 1 and 2. All in all, an effective interpretation of the research problem was given.

3.7 SUMMARY

This chapter examined the research methodology that was employed in the study. A quasi-experimental design was used to gather the data. Since it is impossible to include all the pupils in their huge population, the researcher selected 30 pupils as a sample for this study. The use of a small sample ensures depth analysis using input modelling, runs test Chi-squared test and questionnaires. Inclusion of the data collection, presentation and analysis procedures ensures credible results of the research findings.

Carry, Lewis and Bernard (2007) introduce that, ‘other inverse error” in which 7 + 7 = 1 becomes: x = 1 - 7.

Carry et al (2007) suggest that additive inverse have been employed instead of multiplicative inverse thus referred to as “other inverse error.” Hall (2012) assumes that inverse error might be related to deletion error, where 3x - 3 becomes simply x because as the pupil may see 3 and -3 as inverses and cancels them out. Sleeman (2013) points that pupils view 3x as 3 + x. Hall (2012) suggest that deletion error and other inverse errors may possibly be reduced in frequency by heightened teacher emphasis which reinforces the idea of inverses. Sleeman (2013) proposes that other inverse error is not in the present literature but justifies its inclusion in the study.

The transposing errors are also a related literature. Kieran (1992) points out that, emphasis (on symmetry) is absent in the procedure of transposing. Hall (2012) suggests an evidence of transposing errors in pupils. These problems exist when blindly applying the change side – change sign rule in equations involving denominators such as: \( \frac{x}{2} + 3 = 10 \) It implies that \( x + 3 = 20 \) Greeno (2003) suggest that the transposing errors were dictated in the pilot study. Heibert and Carpenter (2009) point the reason why transposing errors exist. Problems rise due to over generalization of equations such as: \( \frac{x}{2} = 3 \) which follows that? \( x = 6 \). Heibert and Carpenter (2009) found that pupils should construct their own mathematical knowledge when solving algebraic equation rather than receiving it from the teacher or text book. Hart (2013) stresses the fact that, how to construct this knowledge may depend on the people’s previous knowledge and experience as stated by Ausubel. Libinowicz (2008) concludes that, what pupils know about algebra influence the learning of algebra.

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31
A null hypothesis, $H_0$ and alternate hypothesis, $H_1$ were given concerning the distribution of the data under study.

**Procedure 3**

The researcher calculated the expected frequencies using the given distribution.

**Procedure 4**

The calculated Chi-squared value was given.

**Procedure 5**

Decide on the level of significance and decision rule by looking up for critical values in the Chi-squared tables, thus;

Decision rule: 
- Reject $H_0$ if $\chi^2_{cal}>\chi^2(v,a\%)$
- Accept $H_1$ if $\chi^2_{cal}<\chi^2(v,a\%)$

Where $v$ is the degrees of freedom and $a\%$ is the level of significance.

**Procedure 6**

For test statistics calculated, $\chi^2_{cal}$ is the chi-squared value, thus:

$$\chi^2_{cal} = \frac{\text{observed frequency} - \text{expected frequency}}{\text{expected frequency}}$$
The researcher considered the number of runs in truly random sequence; \( b \), say

\[ 3x + 7 = 2x \]

Involves the step

\[ 3x + 7 - 2x = 2x - 2x \quad \text{(deductive)} \]

\[ x + 7 = 0 \quad \text{(Reductive)} \]

Adi (2013) suggests also that;

\[ x + 7 - 7 = 0 - 7 \quad \text{(Deductive)} \]

\[ x = -7 \quad \text{(Reduction)} \]

Hall (2012) proposes that, the deduction-reduction algorithms can be confusing to a beginning algebra pupil. Weaker pupils may get mixed up whose presentation of work tends to be haphazard. Hart (2013), states that teachers teach algorithms and assume once taught they are remembered. Orton (2011 p31) emphasizes, we have ample proof that algorithms are not remembered or they are sometimes remembered in a form that was never taught."

Some literature is based on the Innate High Cognitive demand of algebra. Literature points many psychological processes involved in gaining an understanding of the rules of algebra and operate correctly in accordance with them. Hart (2013) found that only a very small percentage of 13 to 15 year old pupils were able to consider the letter as a generalized number and the majority of the pupils he tested (73% of 13 years, 59% of 14 years and 53% of 15 year old) either treated letters as concrete objects or ignored them. Hart (2013) stresses the fact that, the majority of such pupils do not acquire any real sense of structural aspects of algebra. Hall (2012) suggests that the
Probability distribution was identified to represent the input process. Because data was available, this step began by developing a frequency distribution and a pie chart of the data. Based on the frequency distribution and structural knowledge of the process, family of distribution is chosen.

Procedure 3

Parameters were chosen and they were used to determine a specific distribution family. The available data were the required parameters for the runs test, chi-squared test and for the pie chart were estimated.

Procedure 4

The chosen random distribution and the associated parameters for goodness-of-fit. Goodness-of-fit was evaluated statistical tests. The Chi-squared test was a standard goodness-of-fit test.

All the above procedures were statistically done through simulation using Arena 14.0 under input analyzer. From all the four steps above the distribution data was automatically given out. Chi-squared test was done whose test statistics calculated value and the degrees of freedom were also given.

3.6.2 Runs test

The runs test was used to determine the randomness of the data collected. The hypothesis tested was:

\[ H_0: \text{marks obtained were random.} \]

\[ H_1: \text{marks obtained were not random.} \]

Procedure 1

Negative ramifications in the effective domain of both teacher and pupil could further militate against success.

Confusion between expression and equation is also part of literature to review. Wagner, Rachlin and Jensen (2011) point that pupils frequently attempt to “solve “expression as they add” = 0” to expression the have asked to simplify. Hoyle and Sutherland (2005 p 216) explains “Previous studies have found that many pupils cannot accept an unclosed algebraic expression.

For example;

\[ 2a + a + 3 \]

Gives

\[ = 3a + 3 = 0 \]

\[ = 3a = -3 \]

\[ = a = -1 \]

And \[ x^2 + 5x + 6 \]

\[ = (x + 3)(x + 2) = 0 \]

Either \[ x = -3 \text{or} -2 \]

Hoyle and Sutherland (2005) suggest that, such kind of error indicate an absence of knowledge of the difference in meaning of an expression and an equation. Source of confusion thus, the process of simplifying expression and solving equations are taught at the same time hence reasoning power appears to be tortuous. Kieran (1992) explains two different methods of solving an equation involving fraction as may serve to illustrate such expression/equation confusion:
school premises. However, the researcher calculated the aggregate sum of the form 2 pupils' marks on algebraic test 1 and 2 which he had recorded and entered in his progress record book. Each test was marked out of 25 whose sum serves as the required data. Test 1 was on simple algebraic linear equations and test 2 was on algebraic quadratic equations. Questionnaires provided data used to identify the proportion of the target population which used specific textbooks and multimedia sources. Data was collected in raw form and presented using a pie chart.

3.6 DATA PRESENTATION AND ANALYSIS PLAN
Presenting information in a more readable form helped in to interpret the research findings. Procedures in analysis data shall be backed by the planning and evaluation made in the schemes of work and progress record book for algebraic equations tests 1 and 2. More data was obtained from the questionnaires and presented using a pie chart.

3.6.1 Input modelling
There are four steps used in the development of a useful model of input data.

Procedure 1
Data was collected from the real system of interest. This often required a sustainable time and resource commitment.

Procedure 2

Method A \[
\frac{x}{5} + 4 = 7
\]
\[
\frac{x + 20}{5} = 7
\]
\[
x = 35
\]
\[
x = 15
\]

Method B \[
\frac{x}{5} + 4 = 7
\]
\[
\frac{x}{5} = 7 - 4
\]
\[
x = 3
\]

Wagner et al (2011) argues that, in method A, the left side of the equation has been dealt with as an expression on its own and initially simplified without reference to the right side of the equation. A pupil using method A has deal with an expression with an equation, giving further scope of confusion between expression and equations for example;

Method B \[
\frac{x}{2} + \frac{5x}{3} = 7
\]
In which it is possible to perform the addition first to give; \[
\frac{13x}{6} = 7
\]
but 6 is the lowest common denominator or to begin solving the original equation by multiplying throughout by 6, thus;

\[
6(\frac{x}{2} + \frac{5x}{3}) = 7 \times 6
\]

Hall (2012) suggests that mixture of method A and method B leads to incorrect results. Kieran (1992) states that, “pupils have generally lacked the ability to generate and maintain global review of the features of an algebraic equation that should be attended in deciding upon the next algebraic transformation to be carried out.
2.3 SUMMARY
Chapter 2 helps to define and limit the problem using the historical and associated perspective. It also enables one to relate findings to previous knowledge and how the researcher is able to state how one’s study is added to the current knowledge providing contributory information from other relevant sources to support or refute research arguments. All in the entire chapter enables the researcher to identify gaps in knowledge and unanswered questions. The next chapter will examine the research methodology that will be employed in the study.

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3.4.2 The Chi-squared test
The Chi-squared test was used to test for the distribution of the data using goodness of fit test. The null hypothesis \( H_0 \) states the distribution of the data. The hypothesis tested was:

\[
H_0: \text{marks obtained follow a certain probability distribution.}
\]
\[
H_1: \text{The marks obtained do not follow the given distribution.}
\]

However, the Chi-squared calculated will be compared with the Chi-squared table and with a given number of degrees of freedom and a given significance value. A conclusion will be drawn.

3.4.3 Questionnaires
A questionnaire is a research tool used to probe information buried in people’s mind (Leedy 1997). White (2005) defines a questionnaire as an instrument with open or closed question or statement to which respondents must react. The researcher shall use questionnaires to determine the proportion of the sample which use textbooks and internet sources used to come up with knowledge of algebraic equations. These questionnaires specified the population which used specific textbooks whose title were identified. With questionnaires, large amounts of data were collected from a larger sample on a short period of time.

3.5 DATA COLLECTION PROCEDURE
Data obtained from the progress record book shall be the subject to this research. The researcher seeks the permission of the school head at the Secondary School, in Zaka district, to use the

Runs can be described as above the mean and below the mean. A “+” sign will be used to denote an observation above the mean and a “-” sign will denote an observation below the mean.
3.4 RESEARCH INSTRUMENTS

Gray (1996) defines a research instrument as an attesting device that measures a given phenomenon. The researcher used some statistical tools to analyze the data. These are Chi-squared test and the runs test, which were used to determine the rejection criteria of given testing hypothesis and determine the randomness of the marks obtained by the pupils, respectively. Questionnaires were also used to derive data which answered one of the research question involving sources used by pupils in the learning of algebraic equations.

3.4.1 The Runs test

The Runs test given by Bradley (1968) can be used to test the randomness of the data collected. A run is a sequence of a data with common property. One run and the other starts when this property changes. Runs test can be defined as a series of increasing and decreasing values. The event is always preceded by a no event and the last event is always followed by an event. If N is the number of events in the sequence, the maximum number of events is N-1 and the minimum number of runs is one. The hypothesis tested was:

$H_0$: the number of pupils passing algebraic equations is random.

$H_1$: the number of pupils passing algebraic equations is not random.

In the runs test, it is always the case that the null hypothesis ($H_0$) must support randomness. For $N>20$, the distribution of the number of runs is reasonably approximated by the normal distribution $N(\mu, \sigma^2)$. The approximation can be used to test the randomness of numbers from a generator. In that case, a standardized normal test statistics is given by a “+” or “-” sign depending on whether the numbers are followed by a larger or smaller number.

3.0 INTRODUCTION

This chapter explains how the research is to be conducted, focuses on research design which is a plan for gathering credible data that enables to answer research question and describes the procedures for conducting research. The targeted population, sample size and sampling procedures are outlined in this chapter, aiming to establish a designated criteria for rationalizing the collection of the most accurate and valid information as well as restricting set of participants for which the actual information for the research data is drawn and obtained. Runs test, Chi-squared test and input modelling and questionnaires are the research instruments to be used for they are testing devices for assessing the research problems. These research instruments explain the mathematical tools, statistical and computer packages the researcher used in his research study. Data collection, presentation, analysis and interpretation procedures are clearly discussed in this chapter to gather accurate findings for research objectives, questions and assumptions. However, this chapter is of significance for the clarification of the key variables and their types. It is on the basis of this chapter’s description and justification that the validity of these research findings can be assessed.

3.1 RESEARCH DESIGN

Burns and Grove (2003: 198), defines a research design as a blueprint for conducting a study with maximum control over the factors that may interfere with the validity of the findings. This plan guides the decisions on what, how and from whom to gather and analyses data. In this study the researcher found quasi-experimental design to be the most appropriate approach under study.
Dawes (2010) explains quasi-experimental as an imperial study used to estimate the casual impact of an intervention on its target population and shares similarities with the randomized controlled trial but it lacks the element of random assignment to treatment or control. Also in this quasi-experimental design, one characteristic of an experiment is missing that is control, but randomization hold.

Quasi-experimental design, typically allows, the researcher to control the random assignment to the treatment conditions, by using the same criterion, other than random assignment. Thus the researcher found it more logical to use the mathematical formulae, that is, the Runs test and Chi-squared.

However, quasi-experimental is subject to concerns regarding maximum internal and external validity, because the treatment and control may not be comparable at baseline. With randomization, participants have the same chance of being assigned to the intervention group or comparisons. They are easier to set up than true experimental designs.

3.2 TARGET POPULATION

Best and Khana (1993) defined a population as any group of individuals that has one or more characteristics in common that are of interest to the researcher. Cohen and Manion (2011) posited that “a population refers to the particular universe of persons, objects or events in which a researcher is interested in. From the above definitions population refers to the inhabitants of a place, district or country. That is population can be defined as the total group of persons or objects that meet the designated criteria established. The researcher is targeting three form 2 classes with a total population of 120 pupils. This population is composed of both boys and girls with mixed abilities, whose age ranges from 13-15 years. These pupils provide a pool from which the actual participants are to be obtained and derived.

3.3 SAMPLE SIZE AND SAMPLING PROCEDURES

Coleman (1986) states that a sample is a small group or sub-set drawn from a larger or target population. According to Gray (1996) a sample should have the same distribution of characteristics as the population from which it is selected; hence, it has to be truly representative. Bless (2007) defines sampling as a scientific foundation theory, a technical accounting device to rationalize the collection of information and to choose in an appropriate way, the restricted set of pupils for which the actual information will be drawn. However, sampling involves careful selection of subjects to represent the study population upon which the results will be generated.

The researcher shall use a probabilistic, systematic random sampling procedures to choose a sample size of a quarter of the total targeted population. This implies that 30 pupils were randomly chosen as participants in the research project. To randomly obtain such a sample size, the researcher would use his progress record book to arrange all form 2 pupils’ names in alphabetical order using Microsoft excel 2010, with respect to sum of their marks; they obtained in algebraic equations tests 1 and 2. Each test was marked out of 25 whose aggregate performance provided the required data. He then tossed a coin in such a way that if a head appears, every forth pupil would be included in the sample, starting from the first and top listed pupil. If a tail was to be obtained then every forth pupil would be counted for, starting from the last listed. A head was obtained after tossing a coin. However the researcher included every forth pupil’s mark in the sample, starting from the first listed pupil until the 30th pupil. The chosen sample was the respondents to the questionnaires.