CONCEPTUAL UNDERSTANDING OF THE QUADRATIC FUNCTION CONCEPT IN TEACHERS’ COLLEGES IN ZIMBABWE: A CASE STUDY OF STUDENT TEACHERS AT MUTARE TEACHERS’ COLLEGE, ZIMBABWE

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This project was submitted in partial fulfilment of the requirements of the:

Master of Science Education Degree in Mathematics

Supervisor: Mrs Mutambara
APPROVAL FORM

The undersigned certify that they have read and recommended to BINDURA UNIVERSITY OF SCIENCE EDUCATION for acceptance:

The dissertation entitled “CONCEPTUAL UNDERSTANDING OF THE QUADRATIC FUNCTION CONCEPT IN TEACHERS’ COLLEGES IN ZIMBABWE: A CASE OF STUDENT TEACHERS AT MUTARE TEACHERS’ COLLEGE, ZIMBABWE” submitted by TENDERE JANE in partial fulfilment of the requirements for a degree of Master of Science Education in Mathematics

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DECLARATION

I declare that: Conceptual understanding of the quadratic function concept in secondary teachers’ colleges in Zimbabwe: A case study of student teachers at Mutare Teachers’ college; Zimbabwe is my own work, has not been submitted before for any degree or examination in any other University and that all the sources used or quoted have been indicated and acknowledged as complete references.

Name: TENDERE JANE  Date: 30 September 2016

Signed………………………..
DEDICATION

I dedicate this project to my husband Henry, son Humphrey and daughter Hailey,
who assisted me in various ways. I love you!
ACKNOWLEDGEMENTS

Firstly, I would like to thank God for the life and for protection through and through. My earnest gratitude and appreciation goes to my supervisor Mrs Mutambara for her assistance and guidance for this research project to be a success. I would also like to extend my special thanks to Mrs Kwari, Ms Hwami and the Mutare Teachers’ College Mathematics department members for assistance they gave during the research process. I also wish to extend my gratitude to the Southern African Association of Mathematics, Science and Technology Education (SAARMSTE) members for the help they offered me. Special mention also goes to my family for the financial assistance, encouragement, inspiration and love throughout the duration of this study.

TABLE OF CONTENTS
CHAPTER 1: INTRODUCTION

1.0 Introduction 1
1.1 Background of the study 1
1.2 Statement of the problem 2
1.3 Research Questions 3
1.4 Assumptions of the study 3
1.5 Significance of the study 3
1.6 Limitations 4
1.7 Delimitations 4
1.8 Definition of terms 6
1.9 Summary 6

CHAPTER 2: REVIEW OF RELATED LITERATURE

2.0 Introduction 7
2.1 Theoretical Framework 7
  2.1.1 Description of structures 8
2.2 Students’ Understanding of Quadratic Functions 10
2.3 Linkage between Students’ Mental Constructions and Genetic Decomposition 15
2.4 The Way Understanding a Concept Influences Teaching Methodologies 18
2.5 Summary 19

CHAPTER 3: RESEARCH METHODOLOGY

3.0 Introduction 21
<table>
<thead>
<tr>
<th>Section</th>
<th>Subsection</th>
<th>Pages</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.1</td>
<td>Research Design</td>
<td>22</td>
</tr>
<tr>
<td>3.2</td>
<td>Population of the study</td>
<td>22</td>
</tr>
<tr>
<td>3.3</td>
<td>Sampling frame</td>
<td>23</td>
</tr>
<tr>
<td>3.4</td>
<td>Sampling Procedures</td>
<td>23</td>
</tr>
<tr>
<td>3.5</td>
<td>Samples</td>
<td>24</td>
</tr>
<tr>
<td>3.6</td>
<td>Research Instruments</td>
<td>24</td>
</tr>
<tr>
<td>3.7</td>
<td>The preliminary genetic decomposition of the quadratic function concept</td>
<td>25</td>
</tr>
<tr>
<td>3.8</td>
<td>Ethical considerations</td>
<td>26</td>
</tr>
<tr>
<td>3.9</td>
<td>Validity and reliability of instruments</td>
<td>27</td>
</tr>
<tr>
<td>3.10</td>
<td>Data Collection Procedures.</td>
<td>28</td>
</tr>
<tr>
<td>3.11</td>
<td>Data Presentation and Analysis Plan</td>
<td>28</td>
</tr>
<tr>
<td>3.12</td>
<td>Summary</td>
<td>29</td>
</tr>
</tbody>
</table>

CHAPTER 4: DATA PRESENTATION, ANALYSIS AND DISCUSSION

<table>
<thead>
<tr>
<th>Section</th>
<th>Subsection</th>
<th>Pages</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.0</td>
<td>Introduction</td>
<td>30</td>
</tr>
<tr>
<td>4.1</td>
<td>Presentation, Analysis and Discussion of Written Responses and Interviews</td>
<td>30</td>
</tr>
<tr>
<td>4.1.1</td>
<td>Nature of test items</td>
<td>30</td>
</tr>
<tr>
<td>4.1.2</td>
<td>Understanding of the definition quadratic function (Item1).</td>
<td>32</td>
</tr>
<tr>
<td>4.1.3</td>
<td>Transformations on vertices on parabolas (item 2)</td>
<td>37</td>
</tr>
<tr>
<td>4.1.4</td>
<td>Students’ understanding of item 3</td>
<td>42</td>
</tr>
<tr>
<td>4.1.4.1</td>
<td>Sketching the parabola</td>
<td>42</td>
</tr>
<tr>
<td>4.1.4.2</td>
<td>Vertex form of a quadratic function</td>
<td>42</td>
</tr>
<tr>
<td>4.1.5</td>
<td>Students’ understanding of quadratic word equations</td>
<td>45</td>
</tr>
<tr>
<td>4.2</td>
<td>Responses from the questionnaires</td>
<td>45</td>
</tr>
<tr>
<td>4.2.1</td>
<td>Students’ familiarity with various ways of representing quadratic functions</td>
<td>51</td>
</tr>
<tr>
<td>4.2.2</td>
<td>Use of theories to evaluate learners understanding of quadratic functions</td>
<td>51</td>
</tr>
<tr>
<td>4.2.3</td>
<td>Applicability of quadratic functions in everyday situations</td>
<td>52</td>
</tr>
<tr>
<td>4.2.4</td>
<td>Familiarity with parabolic functions</td>
<td>52</td>
</tr>
<tr>
<td>4.2.5</td>
<td>Teaching approaches</td>
<td>52</td>
</tr>
<tr>
<td>4.2.6</td>
<td>Time allocated for the concept (item17 &amp;18)</td>
<td>53</td>
</tr>
<tr>
<td>4.2.7</td>
<td>Relationship between teachers’ understanding of a concept and teaching</td>
<td>53</td>
</tr>
<tr>
<td>4.3</td>
<td>Summary</td>
<td>53</td>
</tr>
</tbody>
</table>
CHAPTER 5: SUMMARY, CONCLUSIONS AND RECOMMENDATIONS

5.0        Introduction                           54
5.1        Summary of the study and its constraints  54
5.2        Conclusions                           56
      5.2.1        Pre-service teachers’ understanding of quadratic function.  56
      5.2.2        Preliminary genetic decomposition and the students’ responses.  58
5.3        Recommendations                        59
      5.3.1        Affording students more time on Modules containing content taught in schools.  60
      5.3.2        Use instructional methodologies which foster in class            60
      5.3.3        Joining research organizations                                    60
      5.3.4        Misconceptions held by learners                               61
      5.3.5        Relating Mathematics concepts to real world situations         61
      5.3.6        Effectiveness of Exercises and homework                        61

REFERENCES                                           62

APPENDICES
A.        Permission Form                                 70
B.        Consent Form                                    71
C.        Questionnaire for the Lectures                  73
D.        Questionnaire for the Lectures                  77
E.        Questionnaire categories                         78
F.        Lecturers responses on questionnaires            79

LIST OF TABLES

8
<table>
<thead>
<tr>
<th>TABLE NUMBR</th>
<th>CONTENTS</th>
<th>PAGE</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.1</td>
<td>Frequency of responses on item 1</td>
<td>31</td>
</tr>
<tr>
<td>4.2</td>
<td>Frequency of responses on item 2</td>
<td>37</td>
</tr>
<tr>
<td>4.3</td>
<td>Frequency of responses on item 3</td>
<td>41</td>
</tr>
<tr>
<td>4.4</td>
<td>Frequency of responses on item 4</td>
<td>46</td>
</tr>
</tbody>
</table>
## LIST OF FIGURES

<table>
<thead>
<tr>
<th>FIGURE NUMBER</th>
<th>CONTENTS</th>
<th>PAGE</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.1</td>
<td>Effects of varying a ‘a’ on the parabola (where a&gt;0)</td>
<td>12</td>
</tr>
<tr>
<td>2.2</td>
<td>Effects of changing ‘a’ value to a negative</td>
<td>13</td>
</tr>
<tr>
<td>2.3</td>
<td>Effects of varying ‘b’ on a parabola</td>
<td>14</td>
</tr>
<tr>
<td>2.4</td>
<td>Effects of varying ‘c’ on the parabola</td>
<td>15</td>
</tr>
<tr>
<td>3.1</td>
<td>Preliminary Genetic Decomposition for quadratic function concept</td>
<td>25</td>
</tr>
<tr>
<td>4a</td>
<td>Interview excerpt 1</td>
<td>31</td>
</tr>
<tr>
<td>4b</td>
<td>Interview excerpt 2</td>
<td>32</td>
</tr>
<tr>
<td>4c</td>
<td>Extract 1</td>
<td>33</td>
</tr>
<tr>
<td>4d</td>
<td>Interview excerpt 3</td>
<td>33</td>
</tr>
<tr>
<td>4e</td>
<td>Extract 2</td>
<td>34</td>
</tr>
<tr>
<td>4f</td>
<td>Interview excerpt 4</td>
<td>35</td>
</tr>
<tr>
<td>4g</td>
<td>Interview excerpt 5</td>
<td>35</td>
</tr>
<tr>
<td>4h</td>
<td>Extract 3</td>
<td>37</td>
</tr>
<tr>
<td>4i</td>
<td>Interview Excerpt 6</td>
<td>38</td>
</tr>
<tr>
<td>4j</td>
<td>Extract 4</td>
<td>39</td>
</tr>
<tr>
<td>4k</td>
<td>Extract 5</td>
<td>41</td>
</tr>
<tr>
<td>4l</td>
<td>Interview Excerpt 7</td>
<td>42</td>
</tr>
<tr>
<td>4m</td>
<td>Extract 6</td>
<td>42</td>
</tr>
<tr>
<td>4o</td>
<td>Interview Excerpt 8</td>
<td>43</td>
</tr>
<tr>
<td>4p</td>
<td>Extract7</td>
<td>46</td>
</tr>
<tr>
<td>4q</td>
<td>Extract 8</td>
<td>47</td>
</tr>
<tr>
<td>4r</td>
<td>Extract 9</td>
<td>48</td>
</tr>
<tr>
<td>5a</td>
<td>A modified genetic decomposition of the concept; parabola</td>
<td>58</td>
</tr>
<tr>
<td>5b</td>
<td>A modified genetic decomposition of the concept; word problems</td>
<td>59</td>
</tr>
</tbody>
</table>
ABSTRACT

The purpose of this research is to find out the conceptual levels of understanding of pre-service teachers at Mutare teachers’ college. Concerns about high Mathematics failure rate in Zimbabwean Secondary Schools have prompted this investigation into finding out if teachers’ understanding of quadratic function concept could be the cause. These student teachers’ conceptual levels of understanding are explored before they go for Teaching Practice Attachment (TPA). Action-Process-Object-Schema (APOS) theory was used to investigate their conceptual levels. Purposive sampling method was used to select twenty five pre-service teachers from a population of two hundred and five Mathematics pre-service teachers at Mutare Teachers’ College. The same sampling method was also used to select a sample of three lecturers from nine Mathematics lecturers at Mutare Teachers’ college. Written work and follow up interviews were used to solicit information from the students while questionnaires were the research tools for the three Mathematics lecturers. A designed genetic decomposition for quadratic concepts was used as an analysis tool. Majority of the pre-service teachers seemed to operate above the action stage; however, very few have reached the object level. No one was operating at the schematic level. A lot of weaknesses were exhibited by the pre-service teachers in their written work. They seemed to be operating at different conceptual levels of understanding leading to use of different teaching instructions in secondary schools. Findings of the study also revealed that teachers present concepts to pupils the way the understood them so it is important for teachers to understand concepts clearly before they teach others.
CHAPTER 1
INTRODUCTION

1.0 Introduction

Although the focus of Tertiary education policy is mainly rooted in Science, Technology, Engineering and Mathematics (STEM), it is noted with great concern that in secondary schools, subjects like Mathematics and Science are poorly performed. This study mainly focuses on finding out if the quality of teachers produced from teachers’ colleges could be the cause of this situation. In Zimbabwe, this situation is actually a challenge for many because Ordinary level Mathematics is a pre-requisite for one to be enrolled both at primary and secondary teacher training colleges. This is based on the assumption that Mathematics knowledge is essential to all members of society.

It is, however, the purpose of this chapter to present the research process as it unfolds. The background and purpose of this study is given first, research questions, assumptions, significance, limitations and delimitations of the study followed. Later in this chapter, the key terms used in the study with their contextual meanings are outlined.

1.1 Background to the study

Many researches carried out earlier on effective teaching and learning were mainly anchored on the teacher’s knowledge of the subject (Thompson 2004). According to Goodrum, D., Hackling, M., and Rennie, L (2001), teachers are the most important factors to improve students’ learning; therefore, teachers may play a vital role in helping their students’ understanding. In order to find out the level of pre-service teachers’ understanding of concepts and their teaching, one module called; Sets, Functions and Coordinate Geometry was considered. This module is one of the first modules offered to First year Mathematics student teachers at Mutare Teachers’ College. Quadratic function concept is one of the topics entailed in this module. The items covered include parabolas, vertices, intercepts, word problems and vertex form. The researcher happened to have taught this module for seven years and discovered that during lectures, most students were not comfortable with explaining some of these concepts to their colleagues. It was also noted that on average, the students did not perform well at the end of module test implying that they would not have understood the concepts. It is the aim of this research to focus not only how the student teachers
understand quadratic functions but also on how the teachers’ understanding of concepts influence choice of teaching strategies and procedures in the classroom. Pre-service teachers’ misconceptions on the quadratic function concept are also discussed. The actual thought constructions they undergo when they work with quadratic function concept determine whether they have conceptual, procedural or both understanding of a concept. Hapasaalo (2004) defines conceptual knowledge based on a skilful drive on concepts, rules, approaches, etc. The understanding of a Mathematical concept is determined by interactions with existing mental constructions or physical objects to form actions which are internalised to processes which are then encapsulated to form objects. De-encapsulation of objects is a characteristic of this stage where objects are converted back to the process form. Finally, the objects would be organized in schemas (Dubinsky, 1991).

In colleges, lecturers have worked with students from varying Mathematical backgrounds, ranging from little understanding to more sophisticated ways of Mathematical thinking. The student teachers under study have their ages ranging from 19 years to 40 years. This implies that some have since left school while others are fresh from school resulting in different levels of understanding concepts. Despite the differences mentioned above, one issue that seems to be common to all is that students have an Action-Process-Object-Schema (APOS) level at which they are operating as they make their mental constructions on quadratic function concept. Studies suggest that for educators to have pedagogically powerful representation of a topic, they must first have deep understanding of the subject, and this cannot be substituted by anything else (Ma, 1999). This implies that teachers produce better results if they possess a good knowledge base.

It was therefore important for the researcher to investigate on the levels at which the pre-service teachers are operating on quadratic functions before they go for Teaching Practice Attachment (TPA).

1.2 Statement of the problem

Cooney and Wilson (1993) noted that, the emphasis on functions as a representation of real world phenomena and as an important Mathematical structure remains central to contemporary discussions. However, there are many questions about learning quadratic function concept that are still left unanswered. Ma (1999) describes a teacher with profound understanding of concepts as someone who has both conceptual and procedural understanding and able to teach the concepts without challenges. Consequently, Dubinsky and McDonald (2001) suggested that APOS theory can be used as a tool to objectively explain students’ difficulties in a variety of topics in
Mathematics. This research therefore, seeks to explore students’ understanding of quadratic functions using the APOS theory and try to recommend teaching and learning approaches which foster both conceptual and procedural understanding of quadratic function concept. It is therefore the goal of this study to adopt APOS theory in order to gain more insight on the ways in which student teachers understand the quadratic function concept at entry point to their course and to find out how their levels of understanding influence the way they teach.

1.3 Research Questions

The study sought to answer the following questions:

- What is the pre-service teachers’ understanding of the quadratic function concept?
- How do pre-service teachers’ mental constructs of APOS link with precedent genetic decomposition?
- What are student teachers’ weaknesses in the learning of quadratic functions?
- How do pre-service teachers’ understanding of a concept influence the way they teach the concept?

1.4 Assumptions of the study

In conducting the study, the following assumptions were made;

- Go ahead from relevant authorities would be granted to carry out the study.
- The lecturers and the pre-service teachers would assist in bringing up the necessary information on quadratic function concept.
- A sample of 25 pre-service teachers and 3 lecturers would give enough information for the researcher to answer the research questions.
- A student’s performance was determined by one’s cognitive ability and schema development without influence of other contributions of unfair practice.
- The written work, follow up interview and questionnaire used in the research would be able to extract information sought from the respondents.
- The questionnaire and interview respondents would give truthful and sincere answers.
1.5 Significance of the study

The study particularly focuses on quadratic function concept because of its relevance in a variety of situations. Functions have an important place in the area of Mathematics (Cooney & Wilson, 1993; Dreyfus & Eisenberg, 1984; Romberg, Carpenter, & Fennema, 1993; Zaslavsky, 1997). Quadratic function concept is of great importance because it is a component of many algebraic courses which students study. It provides a foundation to the understanding of more complicated types of functions and concepts. (Zaslavsky, 1997). This study is significant to Mathematics educators as they would understand how college students construct Mathematical knowledge when learning the quadratic concept in order to promote both its conceptual and procedural understanding. Mathematics educators also need to understand the mental structures students go through as they learn features of parabolas in geometry. It may also help them plan and scheme effectively. It may be an additional literature to the existing one and used as reference material. More so, it improves the researcher professionally and academically. Results from this research may open up gaps for further research. Findings of this study may influence future classroom instruction in the area of quadratic functions and parabolas. Ideally, changes in pedagogy and curriculum units can be linked to knowledge obtained about the development of students’ understanding of the quadratic function. Teachers, who are interested in using research, can provide their classrooms, and schools, with informed best practices.

1.6 Limitations of the study

The following aspects are limitations in executing the research:

- The study only considered one secondary teachers’ college as a case study because of inadequate time and financial resources. With adequate resources and time, the research could have been done on all fourteen secondary teachers’ college in Zimbabwe. This would increase its applicability and it would be more representative.

- Data collection in the study was confined to the topic “Quadratic function concepts” where APOS was the only theory used to explore the concept. This was so because of lack of time which makes it impossible to consider other topics and theories.
• Limited cooperation from lecturers as they would be busy with lectures and other responsibilities were a hindrance. This was catered for through collecting data during the beginning of the term before the lecturers got involved in many activities.

1.7 Delimitations of the study

The research was carried out in Manicaland province at Mutare Teachers’ College which is a Secondary Teachers’ Training College situated about 5 km from the Mutare city centre along Masvingo road. The study focused on student teachers at the college where the researcher works as a lecturer in the Mathematics department. The participants consisted of first year students specialising in Mathematics as a double major subject. The function concept tends to be very broad but for the purpose of this study, it is confined to the quadratic function concept. It was chosen because it is covered at this level as one of the topics offered in their first year courses: Sets, functions and coordinate geometry. The quadratic function concept under study includes definitions, parabolas and their transformations, vertices, minimum, maximum, and axis of symmetry. On the other hand, other aspects like maximums, minimums, vertices and graph transformations are part of ‘O’ level syllabus which the student teachers would teach in schools. Samples of 25 student teachers and 3 lecturers were used as respondents. This was done for convenience and for financial reasons. Consequently, the results of the study will be peculiar to the college since it was a case study.

1.8 Definition of terms

Operational definitions of key terms were given;

*Student teacher* - one who is studying to be a teacher and as part of the training is closely supervised in a primary or secondary school. Sometimes a student teacher is called practice teacher (Goethals et al., 2004). Kennedy (1999) views a *pre-service teacher* as a college student working under the supervision of tutors and, step by step takes on more classroom administration and instructional responsibilities. In this study, both student teacher and pre-service teacher will be taken to mean somebody who is studying at a teachers’ college.

*Understanding* - refers to the possession of a deeper clear meaning, being able to justify procedures used or state why a process works (Borowski & Borwein, 1989). Thompson (2004) defines
understanding as the ability to apply flexibly what someone has learnt to different problem situations. The ability to effectively communicate what has been learned is another sign of understanding. Learners can explain how and why they moved from one step to the other (Thompson, 2004).

*Quadratic function* - an algebraic expression whose highest power of the unknown is two and it graph is called a parabola (Parent, 2015).

*Conceptual understanding* - refers to learning that involves understanding and interpreting Mathematical concepts and the relationships between the concepts (Hapasaalo, 2004). Star (2006) defined conceptual understanding as a deep and rich knowledge of concepts and their connections.

*Procedural understanding* - instructional knowledge (Hapasaalo, 2004). It is knowledge of steps in order to attain a certain goal (Rittle-Johnson *et al.*, 2001). Star (2006) views procedural knowledge as algorithms known superficially without connections.

1.9 Summary

In this chapter, the background of the study was established. The problem that led to the research was discussed. The purpose and significance of the study were stated. The research questions which the study intended to answer were posed. Assumptions made were discussed, and finally, some contextual definitions for the key terms used in the previous sections and subsequent sections were given. The next chapter highlights literature which is related and relevant to the study of quadratic function concept.
CHAPTER 2
REVIEW OF RELATED LITERATURE

2.0 Introduction

This chapter explores related literature and some relevant studies on the teaching and learning of the quadratic function concepts. It commences with a description of the theory that informs this study, defining the perspective and context of the study. This is followed by a discussion of the function as a concept and how learners understand the quadratic function concept. Thereafter, the linkage between the preliminary genetic decomposition and the student teachers’ constructs follows. Some relevant studies on how teaching and learning of quadratic functions is influenced by one’s understanding of the concept are also considered. Finally, the quadratic function concept misconceptions held by students are cited.

2.1 Theoretical framework

This research is anchored on APOS theory (Dubinsky & McDonald, 2001). The theory informs and guides data collection and analysis (Maharaji, 2013). It describes how Mathematical concepts can be learned and its roots are grounded in the work of Jean Piaget (1964). Dubinsky (1984) initially introduced the crucial aspects of the theory as what goes through one’s mind when trying to learn a Mathematical concept. In this framework, learners mentally construct their understanding of Mathematical concepts. APOS theory makes use of the ACE teaching cycle, which is a pedagogical approach consisting of three components repeated in a cycle. The three components are: Activities, Classroom discussion, and Exercises (ACE). The first step involves activities designed to cultivate students’ development of mental structures but they are performed outside the class (Asiala, et. al., 1996). The second step occurs in class where students reflect back on the activities they were doing under the teacher’s supervision. This is achieved through the different types of interactions which occur as they discuss. Lastly students are given written work in form of homework in order to consolidate the knowledge obtained from the activities and classroom discussions. The implementation of this strategy has proved to be very effective in helping students to make mental constructions required to solve problems related to the topic studied (Dubinsky, 1991).

APOS theory was developed basing on the assumptions that:
• Mathematical knowledge is based on ability to react to a Mathematical problem situation and how solutions are obtained through constructing or reconstructing mental structures in relation to their social context (Dubinsky, 1984).

• An individual does not learn Mathematical concepts directly since the process of learning depends on the possessed mental structures (Dubinsky, 2010). In order to come up with meaning of any Mathematical concept, one applies mental structures (Piaget, 1964).

It is therefore important for an individual to possess mental structures appropriate for a given Mathematical concept in order to foster learning. If appropriate mental structures are not present, then learning the concept may not be possible. The assumptions above imply that the goal for teaching should consist of strategies for helping students construct the right mental structures, and guiding them to apply these structures to construct their understanding of mathematical concepts. The APOS theory emphasises that conceptual formation works in stages and the construction of a complete mental structure operate through, the mental structures: actions, processes, objects, and schemas.

**Action**- is a change of objects which are viewed by individuals as fundamentally external and as requiring instruction which occurs in stages showing how an operation is performed. This change is first conceived as an action when it is a reaction to stimuli which a learner sees as an external. It needs specific instructions and to perform each stage of the transformation clearly (Dubinsky, 2001). For instance, at an action level of understanding, a learner, when working with the quadratic function concept like the vertex would need the formula $x = \frac{-b}{2a}$ in order to locate the vertex then moves on to the process stage.

**Process** - refers to a mental construction that is made by an individual when an action is repeated and reflected upon it. When an individual repeats an action, this action may be interiorised into a mental process (Dubinsky, 2001). Maharaji (2013) refers to a process as a mental structure that performs the same operations as the action, but wholly in the mind of the individual. The learner can just imagine doing the transformation without having to do the steps explicitly. For a student with a process understanding can think of performing the same kind of action without the need of external stimuli. Dubinsky (2001) further says that an individual might just think of performing a process without actually doing it. He can think about reversing it and comparing it with other processes. In continuation of the above example, the process would be that an individual can now find the vertex of any quadratic function, without the need of an explicit formula to follow.
Object - this is a structure from a process where the individual becomes aware of the process as a whole and realises that change can act on it. If the learner can appreciate this and can actually build the changes, then it can be said that the learner has encapsulated the process into a cognitive object (Dubinsky 2001). In continuation from the above example, an individual who is able to compare, relate two vertices of a quadratic function, and create linkage between concepts has encapsulated the process into objects, so for that particular concept, the level of understanding was object level.

Schema - this is an organised and linked logical framework of an individual’s collection of actions, processes, objects and other related schemas. The linkage is due to the fact that it provides an individual with a way of deciding, when presented with a Mathematical problem, whether the schema applies or not (Dubinsky, 2001). It is maintained that this framework occurs in an individual’s mind when presented with problem situation that involves the concept. At this stage, one can apply the concept in real life situations. Maharaji (2013) notes that explanations offered by an APOS analysis are limited to descriptions of the thinking which an individual might be capable of. Maharaji further observes that, it does not necessarily mean that if an individual possesses a certain mental structure, then he or she will apply it in a given situation as this depends on other factors. The main objective of an APOS analysis is to point to possible pedagogical strategies for helping students learn it. The theoretical analysis proposes that, a genetic decomposition is a set of mental constructions that a student might make in order to understand the Mathematical concept being studied (Dubinsky, 2001). Basing on the theoretical framework, the research would like to address issues such as students’ understanding of quadratic function concept, linkage between students’ mental constructions and the genetic decomposition, how the understanding of a concept influences teaching methodologies and finally, the students’ weaknesses in quadratic function concept.

2.2 Students’ Understanding of Quadratic Function concept

Function is described as; a relation in which the first coordinate is never repeated. There is only one output for each input, so each element of the domain has one unique element in the range (Zaslavsky, 1997). In other words, for every $x$ value there can only be one $y$ value. If there was the coordinate point $(2, 1)$ on a function, there could not also be the point $(2, 3)$ due to the fact that the input $x$-value 2 has more than one output $y$-value. Several studies show that learning of the function concept is often facilitated by the early consideration of an action and its interpretation as a process
For the purpose of this study, functions are narrowed down to quadratic function concept. There are three different forms of the quadratic function: the standard form, factored form, and the vertex form. Kotsopoulos (2007) points out that students get confused when quadratic function concept is presented in different ways which they are not used to. Kotsopoulos gives the example of $x^2 + 3x + 7 = x + 4$ being not in standard form and causing students trouble when asked to perform various tasks with it. Ellis and Grinstead (2008) also discovered that learners face a lot of difficulties as they deal with quadratic function concept. These challenges exist in the forms stated below:

- the way they link algebraic, tabular, and graphical representations,
- how they interpret the role of parameters in quadratic functions, and
- the way they wrongly make generalizations from linear functions to quadratic functions.

The structure $y = ax^2 - bx + c$ (where $a \neq 0$ and $a$, $b$ and $c$ are constants) is the standard form of a quadratic function. Other symbolic representations of quadratic functions are the vertex form: $y = a(x - p)^2 + p$ and lastly, the factored form: $y = a(x - x_1)(x-x_2)$ (Ellis and Grinstead, 2008). According to Zaslavsky (1997), some graphical information is revealed by the above mentioned quadratic function forms. The standard form reveals the location of the y-intercept $(0, c)$, the vertex form clearly highlights the turning point of the parabola (vertex) represented by $V(p, q)$, while the factored form indicates the position of the $x$-intercept $(x_1; 0)$ and $(x_2 ; 0)$ (Zaslavsky, 1997).

### 2.2.1 The parabolic effects of the constants $a$, $b$ and $c$ on the equation $y = ax^2 + bx + c$

#### 2.2.1.1 Effects of varying ‘$a$’

Parabola is the name given to a graph associated with quadratic functions. In order to effectively discuss this effect, the function $y = ax^2$ was considered. Altering the values of $a$ causes an up and down stretch of the graph of the function $y = ax^2$ and the same transformation occurs to the function $y = ax^2 + bx + c$ (Chazan, 1992). It was noticed that as the value of $a$ got bigger, the thinner (steeper) the graph became. Also, a decrease in the value of $a$, resulted in the graph becoming shallow (Figure 2.1).
Figure 2.1: The parabolic effect of changing the size of $a$ (where $a$ is non-negative)

 adoption from Ibeawuchi (2010)

Fig 2.1 above dealt with a non-negative $a$. However, effects of changing the value of $a$ to a negative value, (Figure 2.2).
Figure 2.2: The parabolic effect of changing the sign $a$ to negative

*Adopted from Ibeawuchi (2010)*

The graphs maintain the same shape but are reflected about the $x$-axis.

### 2.2.1.2 Effects of varying ‘b’

For the parabola $y = ax^2 + bx + c$, changing the values of $b$ while maintaining the values of $a$ and $c$ unchanged is shown in Figure 2.3.
Figure 2.3: Parabolic effect of changing the values of b, keeping a and c values constant

Adopted from Ibeawuchi (2010)

The graph of \( y = x^2 + bx + 4 \), where \( b \) takes the values: 3; 2; 1; −1; −2 and −3 causes shifts for the range of values changed, however, the parabola remains the same in terms of its shape and direction.

2.2.1.3 Effect of varying ‘c’

Considering the quadratic function expressed in standard form: \( y = ax^2 + bx + c \) altering value of \( c \) results in graph shifts along the y axis by \( c \) units (up if \( c \) is non negative and down if \( c \) is negative) (Figure 2.4).
A transformation in $c$ causes a move on the locus of the turning point along the line $x = -\frac{b}{2a}$. However, the parabola’s shape and direction are maintained. The equation $x = -\frac{b}{2a}$ is very useful since it explains the effects of changing $c$ without including $c$. The line $x = -\frac{b}{2a}$ is the axis of symmetry irrespective of the value of $c$. To obtain the value of $y$ at $x = -\frac{b}{2a}$ (the turning point), there is need to substitute $x = -\frac{b}{2a}$ into the equation $y = ax^2 + bx + c$ to give: $y = a\left(-\frac{b}{2a}\right)^2 + b\left(-\frac{b}{2a}\right) + c$. Any parabola in the standard form, the vertex is $\left(-\frac{b}{2a}; -\frac{b^2}{4a} + c\right)$ (Owens, 1992).

**2.3 Linkage between Students’ Mental Constructions and Genetic Decomposition**

Arnon *et al* (2014) indicate that a genetic decomposition is a tool by which researchers attempt to make sense of how students learn a particular mathematical concept and to explain the reasons behind student difficulties which can exist in the form of a hypothetical model of mental constructions that a student may need to make in order to learn a particular Mathematical concept. These Mathematical concepts may include the vertex, intercepts, graphs and definitions of quadratic functions which are constructed in a learner’s mind. Learners make sense of Mathematical concepts
by building and using certain mental constructions. In APOS theory these are called the stages in the learning of Mathematical concepts (Piaget & Garci, 1983). The genetic decomposition consists of a description of the actions that a student needs to perform on existing mental objects and continues to include explanations of how these actions are interiorized into processes. While at this point, the concept is still seen as something one does. The process is encapsulated into a mental object so as to be conceived as an entity and something that can be transformed. It is entirely possible that a concept may consist of several different Actions, Processes and Objects. Related actions, processes and objects organized into a larger mental structure called a Schema. It is actually a tool that researchers use to explain how students develop, or fail to develop, their understanding of Mathematical concepts. For instance, when given a task, an individual may perform the task correctly, another may have problems, and still another may completely fail. As observed by Springer (2000) cited in when structures involved in learning a particular concept are detailed, a genetic decomposition can help an instructor to uncover sources of difficulty that arise in the learning process. The genetic decomposition may be used to explain variances in performance. The one who succeeds may give evidence of having successfully made one or more of the mental construction(s) called for by the genetic decomposition. The student who shows limited progress may show evidence of having begun to make the construction(s). The student who fails may not have made the construction(s) at all or may give evidence of having been unsuccessful in having made the necessary construction(s) (Arnon et al., 2014).

So in this particular case, the genetic decomposition guides the analysis as well as pointing out on the gaps in the researchers’ understanding of how the concept develops in the mind of the individual. Furthermore, Ndlovu and Brijlall (2015) propounded that genetic decomposition can also help to guide the design of an instruction by providing a description of how a concept might develop in the world of an individual. If the differences in students’ performance cannot be explained by the genetic decomposition that would imply that the genetic decomposition needs revision (Ndlovu & Brijlall, 2015).

2.4 The Way Understanding a Concept Influences Teaching Methodologies

There are different frameworks on how students learn Mathematics. Schwartz (2008) indicated that for an educator to prepare teaching methodologies that foster understanding, it calls for that educator to have conceptual understanding accompanied by the procedural understanding of the particular
concept. Piaget cited in Barker and Czarmocha (2002) talks about conceptual and procedural learning of Mathematics. Procedural knowledge is chiefly concerned with steps or processes followed as one learns a concept, whereas conceptual knowledge is at the other end of the continuum where there is content mastery and easy knowledge transfer. By emphasizing conceptual understanding, there is a chance of reconstructing a process that one may have forgotten. In other words, students have more to work with, not just a procedure (Schwartz, 2008. Therefore, it can be said that Mathematical understanding basically exist as procedural knowledge, conceptual knowledge or both. Learners can display conceptual understanding which can be viewed as an integrated functional grasp of Mathematical ideas. They also understand why a Mathematical idea is important and even the kinds of contexts in which it is useful. In addition to that, they have their knowledge organization coherent making it possible to develop new ideas through linking their existing knowledge with the new information. Conceptual understanding also supports retention of facts. Methods that are learnt with understanding are very connected so that they are easier to remember and can also be easily reconstructed when forgotten. If students are exposed to a method and get to understand it, they are likely to remember it correctly when called upon to. When students have conceptual understanding they can even try to explain a method to themselves and try to correct it if necessary (Jojo, 2011).

The learning process and the process of understanding are ongoing processes that lead to full understanding according to Engelbregcht, Harding and Potgiertter (2010). They further suggest that the process of understanding new Mathematics concepts occurs in levels, in which with every level the student understands deeper. They further claim that for learners to gain deeper understanding, they have to be repeatedly exposed to the intended concept. In addition, they suggest that Mathematics learning consists of two processes which are first time exposure and consolidation. Real learning and understanding comes about by doing more problems of a certain type and this also brings repeated exposure and deeper understanding. Relationships and connections imply that students already possess some knowledge which they use to make sense of a problem in order to come up with new knowledge by making connections with the existing one (Carpenter &Lehrer, 1999). Further, students begin to construct relationships when asked to solve problems that encourage them to use their informal Mathematical ideas in conjunction with prior Mathematical knowledge (Confrey & Smith, 1994; Thompson & Thompson, 1995). Over the last two decades, there have been numerous articles focusing on teachers’ content and pedagogical content knowledge (Ball, Lubienski & Mewborn, 2001; Ball et al., 2008; Silverman & Thompson, 2008). Many of these studies demonstrate that teacher’s instructional practice is influenced by the teacher’s
knowledge (Ball et al., 2001; Borko et al., 1992; Grossman, Wilson, & Shulman, 1989). While secondary Mathematics teachers' content knowledge is usually judged as adequate for teaching, teaching for understanding requires more than simply well-developed content knowledge (Ball & Bass, 2000; Hill & Ball, 2004). In addition, this body of research has revealed the relationship between teachers’ knowledge and their instructional practices is not as simple as initially hypothesized. There is a difference between knowing Mathematics and knowing how to use it in practice. Ball & Bass (2000) propounded the importance of linking teachers’ knowledge of Mathematics to their knowledge of how students develop an understanding of Mathematics concepts. This will then influence their choice of pedagogical practices.

Burke, Erickson, Lott and Obert (2001) assert that there is growing research support for designing classroom instruction that focuses on developing deep knowledge about Mathematics procedures. When instruction is focused only on skillful execution, students develop automated procedural knowledge that is not strongly connected to any conceptual knowledge network (Star, 2000). This instruction resulted in procedures not executed “intelligently” and systematically. Understanding could be achieved, however, if students were given opportunities to develop a framework for understanding appropriate relationships, extended and applied what they knew, reflected on their experiences, and made Mathematical knowledge their own (Carpenter & Lehrer, 1999). They further asserted that (1) when Mathematical knowledge is understood, that knowledge is more easily remembered and more readily applied in a variety of situations (Hiebert & Carpenter, 1992; Kieran, 1992), (2) when a unit of knowledge is part of a well-connected network of Mathematics. APOS theory used in a Maharaj (2010) noted the hypothesis on learning, that an individual does not learn Mathematical concepts directly. This is drawn from Piaget (1964) who asserted that the individual applies mental structures to make sense of a concept. Maharaj (2010) claims that learning is facilitated if the individual possesses mental structures appropriate for a given Mathematical concept, and that if the appropriate mental structures are absent, learning the concept becomes almost impossible. It is therefore important to develop teaching methods that help students develop Mathematical understanding. Vygotsky (1986) noted that the possibilities of genuine education depend both on the knowledge and experience already existing within the student (level of development) as well as on the student’s potential to learn. Engelbrecht, Harding and Potgieter (2010) interpreted this as approaching Mathematics from a conceptual system rather than as a collection of discrete procedures. Students used conceptual understanding of Mathematics when they identified and applied principles, knew and applied facts and definitions and compared and contrasted related concepts. In an effort to understand learners’ understanding of concepts some
researchers would show interview transcripts. This technique was extensively used by Bourdieu, Chamboredon and Parsseron (2000) in the qualitative research which they carried out. In this approach participants’ written work and interview transcripts were reported illustrating how the data was collected.

2.5 Student Teachers’ Weakness in Dealing with Quadratic Functions

In one research, Siyepu (2013) found out that Mathematics students possess procedural knowledge of certain concepts and make a lot of misconceptions. Leinhardt et al., (1990) identified misconceptions as well formulated system of ideas which are incorrect but repeatedly use. Despite many studies on quadratic functions, there is lack of in depth studies on students’ understanding of the vertex of quadratic functions. Ellis and Grinstead (2008) have a small section in their research which relates to finding the vertex of a quadratic function. They looked for students’ connections between coefficients, $a$, $b$ and $c$ in the standard form of a quadratic function and what they believed these coefficients do to the graph of the quadratic function. Many responses were incorrect, and there was an unexpected finding that students believed the coefficient to represent the slope of a quadratic function. Some students believed that the coefficient impacts the location of the vertex, a false understanding, as well as students who believed that the vertex was not impacted by the coefficient; again, this is a wrong understanding. Borgen and Manu (2002) in their article found out that a student who performs well in class may appear to have basic understanding of quadratic function concepts, but in reality, they may not have a conceptual understanding of the concept. They illustrated the idea that even those who perform well in class and appear to have some understanding of quadratic functions in reality may not. By videotaping two students working together on a problem in an effort to determine the fixed point for a quadratic function and to find out if the point is a highest or lowest point, it was evident that even though the paper answer was correct, the students’ understanding of these concepts was weak. According to the study, one student was reliant on the calculator and there was also confusion between the standard form and the vertex form of a quadratic function, which led to improper imaging. Learners encounter many obstacles which hamper their understanding of quadratic functions. Some of these obstacles are conceptual in nature while others are not. Conceptual obstacles are those that have a cognitive nature and can be explained in terms of mathematical structures and concept which can be traced to learners’ earlier learning. Some difficulties are caused by misconceptions which may have been developed as a result of over-generalizing an essential correct conception, or may be due to interferences from everyday knowledge (Leinhardt et al., 1990). However, when there is a difficulty, it does not
necessarily mean that there is a misconception that is responsible for it. Zaslavsky (1997) researched on the misconceptions that cause slow or no understanding of students’ understanding of quadratics. When dealing with the misconceptions, she coined the phrase conceptual obstacles. Conceptual obstacles are “obstacles that have a cognitive nature and that can be explained in terms of the Mathematical structures and concepts that underlie students’ earlier learning experiences. The obstacles which may hinder learners’ understanding of quadratic functions include:

- restricting the graph of quadratic function to the noticeable part of the graph (Zaslavsky, 1999),
- locating a point on the graph by using only eye “measurement” (Zaslavsky, 1999),
- treating two different quadratic functions as if they are equivalent (e.g. treating \( x^2 + 3x - 4 \) the same way as \( 2x^2 + 6x - 8 \)) (Zaslavsky, 1999), and
- resorting greatly to linearity when dealing with nonlinear functions (Zaslavsky, 1999).

Learners could not separate a function from a non-function. They also failed to use notation within the graph of a function itself. Leinhardt et al. (1990) looked at functions and their graphs in general for misconceptions and difficulties, some researchers look at specific functions due to where their focuses are in their investigations. Kotsopoulos (2007) found that secondary students experience many difficulties when factoring quadratics. The difficulties arise due to students being challenged with having to recall basic multiplication facts. Given that the factoring of quadratics is the writing of polynomials as a product of polynomials, students need to have both a strong conceptual understanding of multiplication of polynomials as well as the procedural knowledge to retrieve basic multiplication facts effectively (Kotsopoulos, 2007).

### 2.6 Summary

In this chapter, the researcher provided an overview of literature on functions, their properties, understanding of quadratic function concept, genetic decomposition and some misconceptions held by learners on quadratic functions. Following this discussion, it became clear that there is need for current understanding to facilitate future learning; new knowledge should be incorporated into existing related structures. Key aspects relating to quadratic functions have been explored. The next chapter deals with the research methodology.

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**CHAPTER 3**

**RESEARCH METHODOLOGY**

30
3.0 Introduction

The chapter introduces the research methodology. It is concerned with research design, population and sample, sampling techniques and procedures. Research instruments employed in data collection procedures and data analysis plan are going to be discussed. The preliminary genetic decomposition of concepts in quadratic functions, validity and reliability of instruments as well as some ethical issues are also considered in this chapter.

3.1 Research design

Kumur (2011) defines a research design as a procedural plan for answering research questions validly, objectively, accurately and economically. In simple terms, a research design is a systematic and justified way to answering research questions. Romberg (1992) asserted that when no numbers are used in categorizing, organizing and interpreting the relevant information that have been gathered, then the method is said to be qualitative. This study is qualitative paradigm where a Case study design was employed to explore students’ understanding of the quadratic function concepts. Hartley (2004) views a Case study as a detailed investigation characterized by collection of data within a long period. A concept is usually analyzed within a specific context. This design was selected because it deals with real life context. Hartley also noted that this design has an advantage of building theories. The design can also be undertaken by one researcher who is intensively active, so it does not require a team of researchers to carry out the research. By its nature, qualitative research methodology allows one to use different research strategies to collect data. It also allows for the voice of the participants to be heard. The qualitative paradigm allows the researcher to skillfully devise a tool to probe deeply within the minds or attitudes, feelings and reactions of the respondents. Interviews, document analysis and questionnaires were the tools concerned with exploration of quadratic concepts.

3.2 Population of study

Population can be defined as all the characters under investigation. According to Chiromo (2012), population refers to all the individual units, objects or events that would be considered in a research project. In this study, the population consists of all the student- teachers doing Mathematics as their main study at Mutare Teachers’ College. The population size was three hundred and five (305) student teachers and nine (9) Mathematics lecturers. All the required information was collected
from the students whilst they were in the college before they went out for TPA in order to reduce transport costs.

**3.3 Sampling Frame**

A sampling frame is the complete list of sampling units which is actually the set of all units where the sample is drawn from. For this study, sampling frame was an up-to-date class attendance register which gave the names of all First year Mathematics students and staff list for Mathematics lecturers. These sampling frames were accurate and up-to-date and care was taken to ensure that it was free from omissions or duplications in order to minimise sampling errors.

**3.4 Sampling procedure**

Sampling is a process of selecting a number of individuals for study in a way that they represent the population. The intention of sampling is that the sample accurately represents the whole population and be representative enough so that results are trusted. A non-probability sampling technique called purposive sampling was employed. Creswell (2014) says that in purposive sampling, researchers handpick the case to be included in the sample. In this study, it was based on their willingness to participate. In this way; they build up a sample that is satisfactory to their specific needs. The Sampling procedure was employed to the first year student teachers at Mutare Teachers’ College since they learnt the concept during their first year at college. Lecturers were selected using the purposive sampling method. The criterion was based on their experience of having taught both Ordinary level and Advanced Level Mathematics in the high schools. This was done to ensure that the selected lecturers knew quadratic function concepts from their teaching experience.

**3.5 Sample**

It is usually difficult and expensive to work with the whole population, hence the selection of a sample. Best and Kahn (1993) describe a sample as a portion of the population selected for observation and analysis. A sample is a part of the population under study which is selected from the population. The main objective of drawing a sample is to make inferences about the larger population from the smaller group which should reflect the characteristics of the total population. A total of 25 First year Mathematics students and 3 Mathematics lecturers at Mutare Teachers’ College were purposively selected to constitute the sample.

**3.6 Research Instruments**
The study used a variety of data collection instruments which included questionnaires, interviews and document analysis. Macrae (2007) defines research instruments as testing devices for measuring a given phenomenon, such as paper and pencil test, a questionnaire, interviews and also guidelines for observations. Document analysis was done on the quadratic function concepts written test given to student teachers. A follow up interview on the written work was held. To access information about students on quadratic function concepts, questionnaires were administered to the lecturers.

3.6.1 Document Analysis

The selected sample wrote test on quadratic function concepts after being taught using the ACE method of teaching. These written scripts were analyzed carefully in an effort to find out their understanding level of the concepts. Some follow up interviews were conducted after the marking of the written scripts.

3.6.2 Follow up interviews

An interview is a method of data collection in which one person (an interviewer) asks questions to another person (a respondent) in order to get in-depth information about a topic or concept. Sewell (2002) also defines an interview as an attempt to understand the world from the participants’ point of view, to discover the meaning of the people’s experiences and to uncover their world before giving scientific explanations. Interviews can be conducted in different forms which include face-to-face and telephone (Polit & Beck, 2006). Semi-structured interview approach may be used to follow up key observations in detail to allow opinions or perspectives to emerge freely from the written scripts (Vulliamy et al., 1990). In this study, selected participants were interviewed for clarity and explanations on their written responses. This study employed the semi-structured interviews to solicit more detailed explanation on the student teachers’ written work. Semi-structured interviews are more about flexibility and generation of more useful data. Its nature gave the researcher the opportunity to probe for detailed information since the respondents had the freedom to argue and describe how much explanation to offer (Pathak & Intratat, 2012). The semi-structured interviews captured the students’ insights into their experiences and understandings gained through the written work, and made mental structures suggested in the Genetic Decomposition.

3.6.3 Questionnaires
Questionnaires were administered to 3 Mathematics lecturers in order to collect information that would help explain students’ responses on their understanding of the concepts under discussion. Questionnaires are usually viewed as a more objective research tool that can produce general sable results and evidence of patterns amongst large populations and samples (Harris & Brown, 2010; Kendall, 2008). A semi-structured questionnaire comprises a mixture of closed and open-ended questions. Questionnaires were used to solicit information from lecturers’ on pre-service teachers’ biographic details, knowledge levels, opinions, strategies and challenges in relation to learning of Mathematics (Harris & Brown, 2010).

3.7 The preliminary genetic decomposition of the quadratic function concept

The perspective taken in this study seeks to describe a set of specific mental constructions that a student might make in order to develop an understanding of the quadratic function concept. The result of this analysis is known as a genetic decomposition of the concept. This analysis was influenced by the researcher’s own experience as a Mathematics teacher and lecturer.

![Diagram of quadratic function concept]

- **Schema for quadratic function concept**
  - **Indicators**
  - **Mental constructions**
  - **Competence**
  - **Concept**
    - **Quadratic function**
    - **Parabola**
    - **Action**
      - Graphing using table of values
      - Stating vertex minimum/maximum
      - Stating vertex using formula
      - Describing parabola shape
    - **Process**
      - Defining quadratic function
      - Computation of vertex form
      - Graphing without table of values
      - Understanding the vertex
    - **Object**
      - Understanding word problems involving quadratic concepts
      - Explaining parabolic transformations caused by
3.8 Validity and Reliability of Instruments

The researcher ensured that the instruments chosen were valid and reliable. The validity and reliability of this study depended to a large extent on the appropriateness of the questionnaires and tests. They were structured in such a way that they would solicit the needed information. The process of data collection in qualitative approach entails the use of questionnaires interviews, and written exercises analysis. According to Onwuegbuzie and Johnson (2006) qualitative research has different terms produced to carter for it because the issues of validity are still under debate. The authors argue that because mixed methods research involve combining complementary strengths and non-overlapping weaknesses of quantitative and qualitative research, assessing the validity of findings becomes particularly complex. However, this study gives the general and basic meanings of both validity and reliability. Validity refers to the extent to which a research instrument or tool measures what it is intended to measure (Leedy & Ormrod, 2009). In simple terms, validity is the degree to which a particular instrument is used to ascertain the accuracy, meaningfulness, and credibility of the research study. Reliability focuses on the degree of consistency when a particular technique is applied repeatedly yields the same result each time (Babbie, 2012). In this study, the use of follow up interviews in the data collection process enhanced the opportunity to legitimize the findings and validate the data. This is meant to establish the validity and reliability of the research instruments; questionnaire, document analysis and interview.

3.9 Ethical considerations

Cohen (2007) defines ethics as a matter of principled sensitivity to the right of others. In conducting social science or education research, the wellbeing of all participants must be a priority. All the necessary processes should be adhered to, which protect the physical and mental integrity of the participants, respecting their moral and cultural values as well as their religious and philosophical convictions. Prime concern should be on confidentiality and potential consequences of the study (Beauchamp & Childress, 2001). Ethical considerations in this study focus on acquisition of informed consent, confidentiality, anonymity, avoiding harm to participants, and permission.
3.9.1 Informed consent

To ensure informed consent in this study, the researcher shared with participants what the study is all about and what it seeks to find out. The participants’ informed consent was sought before the commencement of the study to avoid frustrations of both parties in the following processes. The researcher requested participants to sign forms of informed consent before the data collection process.

3.9.2 Confidentiality

Polit & Beck (2006) express confidentiality as the condition where the research participants’ identities are not associated to information provided and are not openly said out. In this study, the researcher clearly assured the participants that no unprofessional ways were used to get information from them. Guarantee was given to the participants that collected data was only used for the reason of the study and nothing else.

3.9.3 Anonymity

This is closely related to the aspect of confidentiality, but empathizes on privacy. Burns & Grove (2005) assert that all participants have the right to privacy, anonymity and confidentiality. The information that each participant shares with the researcher should not be passed on to others in any form, unless specific consent has been given. To ensure this consideration, no real and actual names were used in both data collection and data analysis processes. Instead, fictitious names and codes were employed. The fictitious names were indicated on the written work scripts.

3.9.4 Avoiding harm to participants

Beauchamp & Childress (2001) observe that the data collection method of face-to-face interviews may result in potential consequences for the participants. They explain that for some research participants, a research interview may provide the only opportunity to discuss the identified topic. Although it may not be anticipated, the interview may provoke strong emotional feelings. In light of this observation, participation in this study was voluntary and the participants were given the leeway to withdraw at any time of their choice without suffering any victimization or harm.

3.9.5 Permission
The researcher sought permission from the Ministry of Higher and Tertiary Education, and Mutare Teachers’ College administration. Letters to seek permission were written to these offices. See Appendix A

### 3.10 Procedures of collecting data

A pedagogical teaching strategy called ACE was employed to the whole class of first year Mathematics pre-service teachers in the teaching of the module; Sets, Functions and Coordinate Geometry (Dubinsky 1991). This was done in order to foster mental constructions that foster the understanding of the quadratic function concept. All the three stages of ACE were carefully followed. An exercise was then administered to the 25 student teachers selected as the sample. The exercise consisted of questions covering quadratic function concepts. The concepts included definitions of terms, vertices, parabolas, vertex form, word problems and graph transformations. The exercise was written in 30 minutes time under the researchers’ supervision. The exercise was done individually. After the 30 minutes, all the respondents returned their answer scripts except one who disappeared with it reducing the sample size to twenty four. The researcher marked all the collected scripts. Ten students were sampled out to participate in the follow up interviews basing on their performance. The researcher conducted follow up interviews within 20 to 30 minutes duration. An interview schedule was prepared. The purpose of the interview was explained prior to conducting the interview. The confidentiality of their names and the time needed was also highlighted to them. Fictitious names for the participants have been used to ascertain confidentiality and anonymity. During these interviews, the participants had the chance to clearly explain their written responses. The researcher had to probe them in order to solicit the required information. The interview questions were guided by the students’ responses on their answer scripts.

Questionnaires were distributed to the 3 selected lecturers by the researcher. They consisted of questions addressing the four research questions stated in chapter 1. The lecturers were given 2 days to complete the questionnaires. All the 3 lecturers managed to complete the questionnaires which were personally collected by the researcher.

### 3.11 Data Presentation and Analysis Procedures

Qualitative data from semi-structured interviews was presented in narrative form and tables. Interview conversations were presented as interview excerpts. Written exercises were presented as extracts. Findings from the questionnaires were also presented in the form of tables. Descriptive
statistics was used to analyze data from questionnaires, semi-structured interviews and Document analysis. Data analysis was mainly based on the preliminary genetic decomposition and reviewed related literature in chapter 2.

3.12 Summary

This chapter has given an outline of the research methodology that was employed in carrying out the study. Various research tools like, interviews, questionnaires and document analysis were used in the study. The chapter has also indicated that purposive sampling was used to select the sample. A detailed description of the study populations, samples and instruments was given. Validity and reliability of these instruments were discussed. Lastly, the chapter described data collection and analysis procedures. The next chapter presents the analysis of data collected from the written test, follow up interviews and question.

CHAPTER 4

DATA PRESENTATION, ANALYSIS AND DISCUSSION

4.0 Introduction

This chapter is concerned with the presentation and analysis of data collected using the selected instruments discussed in the previous chapter. Data gathering tools included Document analysis, questionnaires and follow up interviews which were designed in such a way that they gave an insight into pre-service teachers’ understanding of quadratic function concept based on the research
questions cited in chapter 1. A transcription of the students’ written work and follow up interviews on selected tasks is presented. A qualitative analytical approach called dialogue which was supported by Bourdieu, Chamboredon and Parsseron (2000) was used in this research. In the study, interview excerpts illustrated how the data was collected. The selected participants were asked various questions in an effort to extract their mental constructs as they dealt with quadratic function concept. This technique gives readers of the study a clear picture of how the process of data collection occurred.

4.1 Presentation, Analysis and Discussion of Written Responses and Interviews.

The section below exemplifies the actual exercise scripts and how the interviews went on in an effort to provide evidence of student teachers’ APOS level of understanding the quadratic concept (intercepts, vertex, parabolas, and word problems). This analysis is based on the preliminary genetic decomposition designed in order to classify the participants into their APOS theory conception levels.

4.1.1 Nature of test items

The test instrument consisted of four questions with sub-questions. For this study, the sub-questions are presented as items. The first question consisted of recall questions that required students to show their knowledge, such as defining a quadratic function. The second question required a conceptual understanding of parabolas which the students were expected to identify and explain the relationship between parabolas and constants a, b, and c in quadratic function of the form \( ax^2 + bx + c \). This type of question was also used by Chazan (1997) in parabolic transformations. The last question was a mixed bag of higher and low order questions involving; stating vertex, describing the parabola and writing the quadratic function in vertex form. The analysis of students’ responses for each of the questions is illustrated below.

4.1.2 Understanding of the definition of quadratic function (Item1)

Questions 1a and b were concerned with definition of a quadratic function. They were treated as item 1 since question 1b was just a follow up of question 1a. A variety of definitions came out from the students’ written work on question 1a) which required them to define a quadratic function. The table below summarises what the twenty-four students wrote as their definitions of functions.
1 (a) Define a quadratic function.
(b) Explain how you would introduce this concept to a ZJC class.

Table 4.1 Frequencies of scores for item 1 (definition of quadratic function)

<table>
<thead>
<tr>
<th>Category</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Indicator</td>
<td>Equation whose highest power of the unknown power is 2</td>
<td>An algebraic expression whose highest power of the unknown is 2</td>
<td>A function whose highest power of the unknown is 2</td>
<td>A relationship between range and domain such that the highest power of the unknown is 2</td>
</tr>
<tr>
<td>Number of responses</td>
<td>6</td>
<td>6</td>
<td>11</td>
<td>1</td>
</tr>
</tbody>
</table>

Analysing these results, it is evident that the students lacked a clear understanding of the meaning of the term function because they treated the quadratic function in parts, rather than as a complete unit. Most of them interchanged the word function with words like equation and expression. Some just used the same word to define itself, implying that they were not in a position to explain its meaning. These findings depicted a big gap between Zaslavsky (1997)’ proposed function definition and the student teachers’ responses. The idea of mapping one input to one output was not known to the student teachers. They just concentrated on the quadratic aspect only. One student thought that all relationships between range and domain can be called functions, yet a function is a special type of a relationship. However, they all remembered or agreed that the highest power of the unknown is two. The results show that on this concept, the students are operating at action level since they had managed to identify the rule to use during their mental constructions (Dubinsky, 1984). In this
case, the presence of an unknown in the expression with 2 as its highest power is the rule for defining a quadratic function. Question 1b was a follow up question of 1a. The researcher used this question to further probe the students’ understanding of the term quadratic function. Their conceptual understanding of the term was to be measured by the way they explained the concept to pupils as indicated by Shulman (1986) when he said that teachers need effective ways of representing the meaning of concepts they would teach. Some written work extracts and some interview excerpts for selected students are shown below.

**Rum’s understanding of item 1**

The word function was not defined, for example Rum just assumed that any expression with two as the highest power of the unknown is necessarily a quadratic function. This shows that the student used the term function without a clear conceptual meaning of the term. She had no precise conception of the quadratic function. Her answer for question 1b lacked detail but the explanation indicated a smooth transition from linear functions to quadratic functions. With reference to the preliminary genetic decomposition cited in chapter 3, Rum seemed to be operating at the action level because she knew a quadratic function but could not clearly define it. The follow up interview with Rum went on as shown below:

| **Interviewer:** | Is any algebraic expression a function? |
| **Rum:** | Yes, and if the highest power of the unknown is 2, then it is called a quadratic function. |
| **Interviewer:** | How would you clearly link what pupils already know to the new concept of quadratic functions? |
| **Rum:** | It can be viewed as a product of two linear functions, since they would be familiar with linear functions, |
| **Interviewer:** | On question 2, you only explained the effect of making a positive or negative to the graph, how about increasing or decreasing a value? |
| **Rum:** | I do not know the effect on the graph. |
| **Interviewer:** | How about varying the value of c? |
| **Rum:** | C is the y intercept, varying its value shifts the position of the graph along the y axis. |
| **Interviewer:** | What happens to the vertex if the value of b is altered? |
| **Rum:** | Ummm, what I knows is that for negative b, the graph moves to the positive and for positive b, the graph moves to the negative side along the y axis. |

**Figure 4.1:** Rum’s interview excerpt 1
Case 2: Mem’s definition of the term quadratic function

Mem’s meaning of the term function is the same as expression or equation. There is no clear distinction between these words as long as they contained two as the highest power of the unknown. She does not have the conceptual meaning of the word function. She seems to be operating at the action level of APOS. She also presented a smooth move from linear functions to quadratic functions on question 1b (Dubinsky, 1984). This confirms that APOS level of operation is process level because she has an idea of explaining the term to pupils without performing any action externally. An interview was done to confirm this claim. Below is the excerpt of the follow up interview.

| Interviewer: | Are you saying an expression and an equation are the same? |
| Mem:         | They are not the same.                                    |
| Interviewer: | How are they different?                                   |
| Mem:         | An expression does not have an equal sign but an equation has. |
| Interviewer: | So why did you write both terms in your answer?            |
| Mem:         | I had forgotten about the function and I thought the function included both. |
| Interviewer: | So how do you redefine the term?                          |
| Mem:         | It is an equation whose highest power of the unknown is two. |
| Interviewer: | From your answer on question 1b, is there a linkage between linear and quadratic function. |
| Mem:         | I just wanted to derive where quadratic function is coming from; in this case, it is the product of two linear functions. |

Figure 4.2: Mem’s Interview excerpt 2
In Fig4.2, it can be confirmed that Mem is operating at the action level of APOS. This understanding of presenting a quadratic function in terms of an equation is similar to (Eisenberg, 1991)’s findings, where he said the way students present concepts is usually influenced by the way they were initially introduced to concepts. Many teachers introduce concepts in isolation without linking it to other presentations. The ability to repeat what you learnt shows that someone is operating at the action level of APOS.

Far’s understanding of the quadratic function

![Image of handwritten notes]

**Figure 4.3: Far’s understanding of the quadratic function (Extract 1)**

Far defined a function as a relationship between domain and range. The quadratic part is the one he defined as being raised to the power two. The student does not realize that a function is made up of three components, which are domain, range and rule of correspondence. An incomplete description of the concept places Far in the action stage of APOS. His response to question 1b confirms that he is in the action stage of APOS since the explanation is vague. An interview was carried in order to get some clarification on what he wrote.

<table>
<thead>
<tr>
<th><strong>Interviewer:</strong></th>
<th>What do you mean when you say it is the relationship between range and domain?</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Far:</strong></td>
<td>The terms come from algebraic expression where unknown value is the domain, and then the range is what you get after substitution.</td>
</tr>
<tr>
<td><strong>Interviewer:</strong></td>
<td>How else can we define a quadratic function?</td>
</tr>
<tr>
<td><strong>Far:</strong></td>
<td>Yes, it’s more of an equation where the f(x) is represented by y axis.</td>
</tr>
<tr>
<td><strong>Interviewer:</strong></td>
<td>So does it mean that a function is an equation?</td>
</tr>
<tr>
<td><strong>Far:</strong></td>
<td>Yaa function and equation are equal.</td>
</tr>
<tr>
<td><strong>Interviewer:</strong></td>
<td>Besides recap of the previous lesson, how would you introduce this concept to</td>
</tr>
</tbody>
</table>
Figure 4.4: Far’s interview responses (Excerpt 3)

From the interview excerpt, it can be deduced that Far fits in the action stage of the APOS stage because he could not make a clear distinction between a function and an equation.

Figure 4.5: Olly’s work on quadratic functions (Extract 2)

Question 1a required the respondent to define a quadratic function and to explain how it would be introduced to ZJC class. Olly could not define a quadratic function clearly. His response to part 1b also indicated that his introduction would be a weak one. Olly’s response indicates that he has not reached the action level using the genetic decomposition on fig 2 from the chapter 2. He is operating at the pre function level (Dubinsky, 2011).

**Interviewer:** Is a function the same as an equation?

**Olly:** I think they mean the same.

**Interviewer:** 2 is the highest power of which values in the equation?

**Olly:** x value.
**Interviewer:** Do you think this is a motivating introduction to pupils?
**Olly:** Yes, since it is a new concept, the teacher has to define it for them.

**Interviewer:** How is the assumed knowledge of linear equations linked to the new concept?
**Olly:** There is no linkage.

**Interviewer:** Is the gradient of a quadratic function uniform as that of linear function?
**Olly:** There is no difference.

**Figure 4.6: Olly’s Interview (Excerpt 4)**

From this interview, it was noted that Olly had a vague idea about the quadratic function concept. He was not even prepared to go and deliver the concept confidently to ZJC pupils. The learner lacked knowledge of gradients and linear functions. This interview confirms that Olly is operating at the pre-function level, (Breidenbach, Dubinsky, Hawks and Nicholas, 1992).

**4.1.3 Transformations on vertices on parabolas (item 2)**

**Item 2**

Discuss the relation of $a; b; c$ in the quadratic function: $ax^2 + bx + c$ (standard form of a quadratic function). What do these coefficients do the graph vertex?

**Table 4.2: Response frequencies on item 2**

<table>
<thead>
<tr>
<th>Category</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Indicators</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Not Answered or incorrect</td>
<td>7</td>
<td>5</td>
<td>9</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>answers</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Only described $a$ and $c$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>only</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Explained how $a$ affects</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>the vertex</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Explained the effect of</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$a$ and $c$ on the vertex</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Answered the whole question correctly</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
The question required the learner to describe transformations on vertex of parabola after changing values of $a$, $b$, and $c$ on the quadratic function in the form $ax^2 + bx + c$, where $a \neq 0$, $a$, $b$ and $c$ are constants. Most candidates were familiar with the effects of varying $a$ and $c$ only. However, most students focused on changes which occur when the value of $a$ takes positive and negative values. These findings were similar to Chazan (1992)’s who indicated that as the value of $a$ gets bigger, the parabola becomes steeper and as the value of $a$ gets smaller, the fatter the graph. From Chazan’s work, it is clear that use of diagrams could have facilitated the explanations of the transformations on these parabolas. However, the concept of increasing or decreasing the $a$ value was not discussed by many students. For $b$ most students had no idea on its effect on the vertex of the parabola. Students’ written work and interview excerpts were highlighted.

**Rum’s understanding of a parabola**

On question 2, Rum clearly explained the effect making “a” positive and also making it negative on the vertex. She was silent about $b$ and $c$. A follow up interview was done in order to find out why she left some of these parts unanswered and to find out the APOS level she is operating at. Fig 4.7 highlights how the interview went on.

---

**Interviewer:** What is the name given to the graph of a quadratic function?

**Rum:** It is called a parabola.

**Interviewer:** What happens to a parabola if we vary “a” value on a quadratic function in the form $ax^2 + bx + c$?

**Rum:** If the $a$ is positive graph will be cup shaped and if the value of $a$ is negative, the graph will be cap shaped.

**Interviewer:** What if we increase or decrease the value of $a$?

**Rum:** I don’t know.

**Interviewer:** How about varying the value of $b$ keeping $a$ and $c$ constant.

**Rum:** Negative $b$ shifts the graph to the right along the x axis and positive $b$ shifts the graph to the left side along the x axis.

**Interviewer:** What is $c$?

**Rum:** It is the y intercept.

**Interviewer:** If we change the intercept what happens to the graph?

**Rum:** If $c$ is positive, the graph moves upwards but if $c$ is negative the graph moves downwards.

---

**Figure 4.7:** Rum’s Interview (Excerpt 5)
From Fig 4.7, it can be concluded that Rum failed to present her answers clearly on paper but from the interview, clearly explained herself. Rum is operating at the object level since she could picture the graph transformations correctly without attempting to draw the graph. According to Dubinsky (2001), Rum is operating at the object level where she takes the process as a whole with reversible changes and she created clear linkages between the concepts: intercepts parabolas and vertices.

Figure 4.8: Olly’s understanding of concepts on Item 2 (Extract 3)

The question required the candidates to be familiar with the standard form of a quadratic function $a x^2 + b x + c$, explaining the effects of varying the constants a, b, c to the graph vertex. Olly could not link the constants with the parabola. The student teacher failed to come up with actions, rules or procedures to form an action on solving the problem. The student teacher is operating at this level which is below action level but the preliminary genetic decomposition on fig 2 failed to allocate a level for Olly. To confirm this, the researcher carried out an interview, (fig 4.6).
**Interviewer:** Could you explain the effects of varying the constants; $a$, $b$, $c$ on the vertex of a parabola?

**Olly:** What I remember is that “$a$” should never be zero for one to be able to draw the graph.

**Interviewer:** How about $b$ and $c$?

**Olly:** $b$ is the gradient of the graph and $c$ is the y intercept.

**Interviewer:** Is the gradient of a parabola uniform?

**Olly:** It is the same as the gradient of a linear function.

---

**Figure 4.9: Olly’s interview (Excerpt 6)**

From the interview, it is clear that Olly is operating at pre-action level because there was no understanding of the question requirements. He just presented what he remembered which was not even correct. A misconception was also exhibited when he stated that $b$ is the gradient of the quadratic function. These findings were similar to what Grinstead (2008) found from researches carried out, Grinstead said many students related the coefficients $a$, $b$, $c$ to the slope of the quadratic graph, which is incorrect. On the effect of varying $b$, during his study, Ibeawuchi (2010) proposed the idea of drawing a quadratic graph with $a$ and $c$ remaining constant while $b$ changes. It was however noted that no one of the respondents tried the problem in a similar way.

---

**Figure 4.10: Student Y’s work on the parabola (Extract 4)**

Instead of explaining the effects of varying the constants $a$, $b$, $c$ on the vertex, the student talked of roots instead. During the follow up interview, the student indicated that this was all he remembered on the constants $a$, $b$, $c$. The student is operating at the action level where he memorized some formulae and tried to use them, but in the wrong place. The question was not conceptualized.

---

**4.1.4 Students’ understanding of item 3**
Item 3

3) Given the following function: \( f(x) = x^2 + 4x + 4; \)
   a) draw and its graph,
   b) The vertex is (---, ---) and is a minimum or maximum?
   c) Write the quadratic function in vertex form.
   d) How would you explain the vertex concept in relation to axis of Symmetry to a ZJC class?

Table 4.3: Frequencies of item 3 scores

<table>
<thead>
<tr>
<th>Category</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sketch the graph</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Present function in vertex form’</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Explain vertex in terms of symmetry</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Concavity</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Number of responses</td>
<td>24</td>
<td>6</td>
<td>6</td>
<td>12</td>
</tr>
</tbody>
</table>

Question 3a required the learner to draw the parabola and say if it is concave up or concave down. Most students got confused by the terms resulting in half of the students getting the answers correct and the answer half getting wrong answers. From the follow up interviews it was noted that the students were familiar with the terms: concave and convex not concave up or down. The half which got it wrong did not construct the action conception of the concept. They are operating at pre-function level. The other half has reached the action level of APOS.

4.1.4.1 Sketching the parabola

All the respondents managed to draw the graph of the parabola; however, two students created tables of values in order to draw the graph. Their conception of graphing is rooted on table of values. Almost all the student teachers exhibited the process level except for the two who could nothing without generated values.

4.1.4.2 Vertex form of a quadratic function
From the twenty-four participants, only six had an idea of the meaning of vertex form. Majority of the respondents confused it with the factorized form. Others had no idea of finding the vertex form. Some used the method of completing the square to express the function in vertex form, while others got the answer correct but did not show the working. When dealing with the axis of symmetry, the participants viewed it in two ways:

- as a line that bisects the vertex or the whole graph as a whole;
- as a number derived from the formula $-\frac{b}{2a}$

As for the vertex, the participants considered it to be the highest or the lowest coordinate pair $(x, y)$ depending on which direction the parabola was orientated. Below are more students’ test extracts and the interview excerpts:

**Figure 4.11: Rea’s work on item 3 (Extract 5)**

The student shows both procedural and conceptual understanding of the concepts in item 3. The student seems to have encapsulated the process level to object level of understanding. An interview was done to verify his written work. The interview excerpt is shown below:

**Interviewer:** Are you familiar with the two terms: concave up or concave down?
Rea: What I know is that concave up is the same thing as cup shaped and concave down refers to a cap shaped graph.

Interviewer: How did you find the vertex?
Rea: I just read its coordinate from the graph.
Interviewer: How did you find the vertex form of this function?
Rea: After factorizing it, discovered that it was already in the required form.

Figure 4.12: Rea’s Interview (Excerpt 7)

The interview confirmed that the student’s knowledge is not based on memorizing formulae but conceptual understanding of the concepts. From the indicators on the preliminary genetic decomposition, he can be categorized into object level of the APOS theory.

Figure 4.13: Ten’s understanding of item 3(Extract 6)

Ten did not draw the graph so it became difficult for him to tell that the vertex is a minimum. Most students do not enjoy graph work, which a weakness for most Mathematics student teachers. This weakness was also highlighted by lecturers in the questionnaires they answered. This is also supported by Ellis and Grinstead (2008) who said that students have problems in linking algebraic statements with graphical representations. He got a wrong answer on that second part because his first part was incorrect. Ten did not even state the coordinates of the vertex. The stages for completing the square method were incorrectly done. He failed to attempt the last part of the item. According to (Dubinsky and Harel 1992), Ten is operating at Pre-function level. This level is not contained in the preliminary genetic decomposition, hence the need to come up with a modified genetic decomposition as substantiated by Ndlovu and Brijlall (2015). Ndlovu and Brijlall propounded that if differences in student performance cannot be explained by the genetic
decomposition, then that would be implying that the genetic decomposition needs revision. An interview was done to get some explanations on Ten’s written work.

| **Interviewer:** Why didn’t you draw the graph of the function? |
| **Ten:** I did not think it was necessary. |
| **Interviewer:** What do you understand by the term vertex? |
| **Ten:** It is the turning point of a graph. |
| **Interviewer:** How did you come up with the vertex form? |
| **Ten:** I don’t remember, I just tried to play around with the figures. |
| **Interviewer:** Why did you leave part d? |
| **Ten:** I do not find the connection between the vertex and the axis of symmetry of a graph. |

Figure 4.14: Ten’s Interview (Excerpt 8)

The interview excerpt indicates that there are no rules or algorithms used when he dealt with the question. The preliminary genetic decomposition in fig 2 could not explain Ten’s level. Ten could be operating at a level below the action level which Dubinsky & Harel (1992) named pre-function level as stated in their findings. According to Ball & Bass (2000), Ten may have difficulties in teaching the concept which is difficult for him to understand. This situation then calls for revision of the preliminary genetic decomposition in order to come up with a modified one. This is in agreement with what Ndlovu and Brijlall (2015) propounded in their research on matrix algebra.

4.1.5 Students’ understanding of quadratic word equations

**Item 4**

4) The height h, in metres of an object above the ground is given by \( h = 16t^2 - 64t + 19 \), where t is time in seconds and it is given that \( t \geq 0.5 \).

Find the time it takes the object to strike the ground and find the minimum/maximum height of the object.
Table 4.4: Frequencies of scores on item 4

<table>
<thead>
<tr>
<th>Indicator</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>No attempt made</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>21</td>
<td>8</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>Attempted but got a wrong answer</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Used quadratic formula wrongly</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Used quadratic formula and got first part</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Substituted a wrong t to find the min height of the object</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Answered the whole question correctly</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Answered the whole question but was not sure of the second part</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The question required the respondents to calculate the time taken by the object to strike the ground and the minimum height of the object. One student did not even attempt to answer the question. A follow up interview on this student revealed that he had no idea of how to go about it. Another student used an incorrect quadratic formula in the process of trying to answer the first part of the item. The student used $2ac$ instead of $2a$ as the denominator of the quadratic formula. According to APOS, these three students did not even fit in the action level but below the action level. The students are said to be operating at pre-function level as propounded by Dubinsky and Harel (1992). Twenty-one students were operating at action level since they were able to use the quadratic formula correctly to find the time taken by the object to strike the ground, answering the first part only. On the second part, eight students failed to find the $x$ value of the turning point of the graph but simply substituted the value they got in part (a) which was not correct. Only one respondent got the whole question right. He clearly understood what was going on even without constructing the graph. The student teacher performed a new Mathematical operation of the quadratic concepts. According to APOS theory, this student is operating at the process level. A student operating at this stage can...
represent the solution using different forms. They can also explain and justify use of a chosen method used. Reviewed literature indicates that this student has greater chances of adopting methodologies which foster conceptual understanding of concepts as propounded by Ball & Bass (2000). However, two students seemed to be operating at the process level. They correctly used the formula $\frac{-b}{2a}$ to find the x value of the vertex point but had a weakness of failing to complete their calculation of the y value because they were no longer comfortable with a negative y value they got.

![Figure 4.15: Extract 7(Tam’s written work)](image)

Tam’s work proves that he can actually work with problems in different forms, and has encapsulated the processes into a cognitive object (Dubinsky 2001). From these findings, the preliminary genetic decomposition fits well with the exhibited mental constructions.
Tam exhibited conceptual understanding of the quadratic function concept. No weaknesses shown in his work. This implies that if he is to teach this concept, he would use methodologies which promote conceptual understanding since there is a linkage between one’s content knowledge and pedagogical knowledge as substantiated by (Ball & Bass, 2000; Hill & Ball, 2004). The follow up interview outcome also supports the analysis above.
Figure 4.17: Extract 8 (Rub’s written work)

The follow up interview results with Rub revealed that he clearly understood in his mind the requirements of the question but he was not sure whether he was still in the right tract since he was about to get a negative answer. This showed that he had interiorized the actions into processes, so he was operating at process level where the actions actually occur in the mind. (Dubinsky 2001).

The only weakness which Rub showed was lack of confidence in what she was doing. This may even affect her selection of teaching methodology as propounded by (Ball et al., 2001; Borko et al., 1992; Grossman, Wilson, & Shulman, 1989). The preliminary genetic decomposition managed to link well with these findings.
There is clear evidence that Bri is operating at the action level, where the quadratic formula was correctly used to get the values of $t$. The second part of the question was difficult for him that he ended up putting down the answer without showing working. However, interview confirmed that this answer ($-157, 16$) was obtained by substituting 3, 67(obtained in part a) into the original quadratic function, which is a misconception. This is similar to what Leinhardt et al (1990) found out. Misconceptions are identified as incorrect features of student’s knowledge that are repeatable but not simply an error.

### 4.2 Responses from the questionnaires

A questionnaire is one of the instruments used for data collection. It was administered to three mathematics lecturers who were selected conveniently. The questionnaire consisted of twenty questions which were sub divided into three sections: understanding of quadratic functions (items 1 -5), applicability of quadratic functions to everyday situations (items 6-15) and teaching approaches (items 16-20) as shown in Appendix E

Out of the three lecturers, two agreed that students’ understanding of quadratic functions is procedural. This is supported by students’ failure to apply the concepts to different problems and
often they tend to give answers as per procedure. However, one lecturer said that there are some students who exhibit conceptual understanding of quadratic functions as they respond to questions concerned with quadratics. The findings suggest that most pre-service teachers are operating at the action level with very few floating above this level.

4.2.1 Student teachers’ familiarity with different ways of representing quadratic function.

One lecturer strongly agreed that student teachers are very familiar with quadratic function concepts from their secondary school and another lecturer also agreed that students are familiar with these concepts as shown by their work when given time to work on quadratic functions. Written exercise was given to the pre-service teachers to show their mental constructions as they work with quadratic functions. For a student to get new mathematical meaning about a concept, he or she would need to construct mental representations of experiences that are of relevance to that particular concept. According to Dubinsky (1991), the student needs actions, processes, objects and schemas (APOS) as mental constructions.

4.2.2 Use of theories to evaluate learners’ understanding of quadratic functions.

All the three lecturers concurred that appropriate theories can used to find the level at which the pre-service teachers are operating as they deal with quadratic functions. They further agreed that the learners’ understanding is mainly based on different factors like the way they were taught. This research chose the APOS theory in an effort to find out their level of understanding of quadratics because of its suitability to “analyze the knowledge that students display when solving a specific activity at a particular moment in time. This is in agreement with what Polanyi, Trigueros, Preciado, & Lozano (2009) found out.

4.2.3 Applicability of quadratic functions in everyday situations

Borgen and Manu (2002) illustrated that even the students who appear to have understood the concepts may not be in a position to relate it to the other scenarios in the real world. The ability to relate a concept to other situations shows a conceptual understanding of the concept. Hebert and Lefevre (1986) referred conceptual understanding as “a connected web of knowledge. It is of importance to make use of teaching strategies that help students to develop mathematical understanding. (Brijlall & Maharaj, 2009)
4.2.4 Familiarity with parabolic functions

Two lecturers stated that this idea started at secondary school and at this level, they seem to have mastered it. However, the other lecturer said that not all the students are in this category but very few. It is clear from the questionnaire that the students are not familiar with all the characteristics of the parabolic function. The three lecturers share the same view on this issue. Lecturers view the idea of linking parabolic functions to everyday situations as a difficult task for students. They said the learners consider the concept as abstract hence no relationship found between this concept and reality. Rittle-Johnson and Alibali (1999) have found that when students learn procedural knowledge only, they have a harder time transferring the information; yet when students possess conceptual knowledge; it is then reinforced by procedural practice. Lack of conceptual understanding keeps students’ mental structures at action level. These actions rarely interiorise into processes.

4.2.5 Teaching approaches

It is thought that if one understands how students think when engaging in Mathematics activities then one might be able to improve on ways of making the learning of Mathematics more meaningful. In the Mathematics world, there is consensus that students’ ways of thinking should be taken into account when planning instruction and that teachers should choose or design sequences of lessons for use and discussion in class (Tsamir, 2003)

Items 16 and 20 are related. The two were about learner centred approaches. Item 16: Some lecturers do make lectures learner oriented. All the lecturers agreed with the statement that there is an effort being made by lecturers to make learner centred lectures. On item 20, the three lecturers also share the same view that learner centred approaches may enhance understanding of concepts taught. This is shown on Appendix E.

Although educators’ experience and qualifications were not used as indicators of educators’ knowledge in this study, it is however important to note that the three educators had at least a degree in education and at least fourteen years lecturing experience. Shulman, (1986) says an educator with both synthetic and substantive knowledge will not only be capable of defining for learners the acceptable truths in a domain but will also be able to explain why it is worth knowing and how it relates to other disciplines, both in theory and in practice. The findings clearly show that there is a
clear relationship between teachers understanding of the concept and them way he or she teaches. All the lecturers strongly agreed with the statement.

4.2.6 Time allocated for the concept (item17 &18)

The lecturers generally felt that time allocated for this course on the timetable was adequate but a lot of activities pop up during the term leading to incompletion of the syllabus or skeletal coverage of some concepts. One lecturer explained that given enough time to teach parabolic functions, it is possible to push all the students to schema level. It was clarified that students normally show action conception level because they would do these concepts just to prepare for the exams. It also came out of the findings that it is important for educators to explore all teaching methods that enhance students’ understanding of concepts if time permits.

4.2.7 Relationship between teachers’ understanding of a concept and the way he/she teaches it (Item 19)

There was an agreement that teachers teach the way they understand a topic. This implies that teachers present what they know and mastery of the subject instils confidence. This idea is supported by many studies that demonstrate that a teacher’s instructional practice is influenced by the teacher’s knowledge (Ball et al., 2001; Borko et al., 1992; Grossman, Wilson, & Shulman, 1989).

4.3 Summary

This chapter gave explanations of how students in the first year constructed concepts of the quadratic function concepts. Analysis presented in tables and students’ written work extracts served to explore the conceptual understanding of these concepts using APOS and the stages on which mental constructs on the quadratic function concepts were made using the genetic decomposition. Follow up interviews were used to check on the answers on their written scripts. The next chapter concludes the study by discussing the findings, thoughts, and recommendations.
SUMMARY, CONCLUSIONS AND RECOMMENDATIONS

5.0 Introduction

The chapter gives a summary of the study including its constraints. This is followed by the conclusions which are guided by the research questions stated in chapter 1. Finally, the recommendations that can be adopted and implemented are captured. The whole chapter is concerned with answering research questions stated below;

- What is the pre-service teachers’ understanding of the quadratic function concept?
- How do pre service teachers mental constructs of APOS link with preliminary genetic decomposition?
- What are student teachers’ weaknesses on the quadratic function concepts?
- How do the teachers’ understanding of a concept influence the way they teach the concept?

5.1 Summary of the study and its constraints

The aim of the study was to explore students’ understanding of the quadratic function concept using APOS theory. This was a case study of Mutare teachers’ college Mathematics pre-service teachers. The background to the research study was rooted on poor Mathematics performance by pupils in the secondary schools, with particular reference to quadratic function concept. The researcher was then motivated to find out if the poor performance resulted from the teachers’ levels of understanding concepts. It was also the researcher’s aim to find out whether teaching styles could have been influenced by the level of understanding of the concepts. Action, Process, Object and Schema (APOS) theory was the theoretical framework. It was introduced by Dubinsky (1984) in an effort to study what will be happening in someone’s mind when learning concepts. Four research questions guided this research.

Literature has shown that the learning and understanding of concept is a process which is ongoing (Engelbregcht, Harding and Potgieter, 2010). They believe that learning and understanding of quadratic function concept occurs in levels as learners are continually exposed to the concept. Arnon et al., (2014) suggested the use of an initial genetic decomposition to find out mental constructions that a student may need to make in order to learn a particular mathematical concept. A revised one may be devised depending on the mental constructions made by participants under study. Maharaj (2010) claims that learning becomes possible if an individual possesses mental structures appropriate for a given Mathematical concept, otherwise, without any mental constructions learning
the concept becomes almost impossible. Educators were encouraged to employ teaching strategies which foster both procedural and conceptual understanding of concepts (Star, 2000). Some learners’ weaknesses in quadratic function concepts were identified as misconceptions which took different forms (Leinhardt et al., 1990).

The research design employed in the study is explorative case study which was qualitative in nature. A mixed paradigm could have helped to improve on research validation. This paradigm makes use of both qualitative and quantitative paradigms. Mixed research methods involve combining complementary strengths and non-overlapping weaknesses of quantitative and qualitative research, assessing the validity of findings becomes particularly complex. Also, the mixed methods approach eliminates biases that might come due to use of a single method because they tend to neutralize each other (Creswell, 2014). Two hundred and five was the population from which a sample of twenty-five was selected. The study was limited to the Mathematics students in the college, those on TPA were not considered because they were not easily accessible. Purposive sampling was the non-probability sampling method used to choose both samples: students and lecturers. Questionnaires, written tasks and follow up interviews were the data collecting instruments used. Major constrain was wrong timing for data collection. The follow up interviews were meant to validate what the students had written down. The levels at which the students operated were determined by the preliminary genetic decomposition which was subjective to the researchers’ experience. Ethical considerations were made in the process of data collection, presentation and analysis. The major constrain faced was time to carry out the research interviews. It was not easy for the respondents to find time for the interviews since they had a lot of work to be done. This created a situation where some interviews were conducted in the morning, during lunch time and sometimes during weekends.

Data collected was presented in the form of tables, which were analysed and compared. Students’ understanding of quadratic functions exhibited from the written exercise and follow up interviews. The results obtained from the written tests given and interviews all agreed on the fact that mental constructions for students vary among students. The mental constructs displayed by the students did not perfectly link to the preliminary genetic decomposition in figure 3.1. Lectures’ responses from the questionnaires showed that educators’ understanding of concepts influences the way instruction is delivered (Ball et al., 2001; Borko et al., 1992; Grossman, Wilson, & Shulman, 1989). Zaslavsky (1997) found out that students have a range of weaknesses which emanate from misconceptions.
5.2 Conclusions

This section has provided answers to the four research questions. Results reveal that students operate at different levels of APOS on certain concepts. Generally, most student teachers were at the action level. Very few interiorised the action level to the processes stage. Two student teachers managed to go through encapsulations of processes to form object. No one reached the schema stage of APOS on the quadratic function concept.

5.2.1 Pre-service teachers’ understanding of quadratic function

The main aspects considered in the written work, interviews and questionnaires were: i) axis of symmetry, ii) vertex, iii) graph orientation, iv) y-intercept, v) graph transformations, vi) maximum or minimum point, and vii) vertex form. It was observed that all students attempted the question on definition of a quadratic function but their definitions were incomplete. All responses acknowledged the presence of an unknown in a statement with the highest power as two. However the term function was given different meanings like equations, and expressions relationships. No one of the responses highlighted the proper meaning of a function. The fact that there was an action of identifying the highest power of the unknown as two implies that the students were operating at an action level of APOS. There were very few respondents who provided a clear introduction of the concept to ZJC pupils on question 1b. These were considered as operating at the process level because there was evidence of interiorising the quadratic aspect through the ability to link the quadratic concept to the linear concepts as shown in Extract.

For Question 2, there were 7 responses that were categorized as indicative of pre-action-level engagement because of two reasons: either not attempting the question or answering it incorrectly. Two of them did not even attempt the question. The other 5 responses were incorrect. They had no idea of what would happen to the vertex of the parabola when values of constants a, b, and c changed on the quadratic standard equation. However, there were some of student teachers who did not understand the question requirements, they went on to define the coefficients a and c rather than describing the effect of varying these coefficients on the parabolic vertex. There were 11 responses where the effects of varying a and c were correctly presented. These 11 were silent about the impact of adjusting b value on the vertex of the parabola. Follow up interviews confirmed that they did not know anything concerning b. However only one response had a clear answer for this question and
the follow up interviews confirmed that the understanding of the concept had interiorised and the
person was aware of the processes and transformations which could occur to parabolas.

For question 3, on graph sketching, all students managed to follow all the procedures to come up
with the parabolas. This is a characteristic of the action level, so all have reached the action level.
To determine the vertex of the graph, twenty students used the graph they had drawn to find the
vertex. Interiorisation of action into process was displayed by these students as they were able to
extract information from the graph they had drawn in order to state the vertex of the graph. They
could see the connection between the two aspects; parabola and vertex. However, four students used
the formula \( \frac{-b}{2a} \) to find the vertex coordinates of the graph. The 4 students failed to make meaning
from procedures and processes of parabolas. It therefore implies that these four students had
procedural knowledge only on graph vertices and this is in agreement with Siyepu (2013)’s findings
which revealed that students exhibited procedural mathematics knowledge in their responses. The
vertex form aspect was only familiar to six students out of twenty-four. The other eighteen went
blank since they had no idea of the meaning of the phrase. This means they had not constructed the
terms cognitively. Findell (2006) pointed out that If one is to remember meaning and use of a
concept, there has to be some cognitive structures constructed around its name. This scenario was
difficult to use APOS to determine the level at which these students are operating. However, the six
managed to use the method of completing the square to express the function in vertex form. This
places them in the action level since they were good at performing actions as they worked with
formulae. From the six, one student proved to be operating at object level because he could verify
these actions verbally during the follow up interview. Majority of students in item 4 are operating
at action level because they could use the quadratic formula to find the time it takes for the object
to fall to the ground. This action was easily carried out.

5.2.2 The extent to which the preliminary genetic decomposition explained the pre-service
teachers’ responses

It was noted from data analysis that the preliminary genetic decomposition failed to accommodate
all students’ responses. Some responses were not clearly stated, hence the need for follow up
interviews in order to probe and understand their mental constructions based on the written
responses. It was noted that the preliminary genetic decomposition needed a refinement in order to
accommodate these gaps. This lead to a modified genetic decomposition presented below.
Figure 5.1: A modified genetic decomposition of the concept; parabola
5.2.3 The students’ weaknesses on quadratic functions

It was observed that these pre-service teachers had a lot of misconceptions and errors. Some incorrectly took it as the gradient of the quadratic function, which is a misconception from the linear functions. This is in agreement with what Zaslavsky (1997) and Ellis and Grinstead (2008) found out in their studies that in most cases students tend to treat linear and quadratic functions in the same manner. In this study, most students thought that the value of c in the quadratic function $y = ax^2 + bx + c$ and linear function $y = ax + c$ is gradient. This is actually a misconception which is a sign of inadequate mental constructions leading to lack of conceptual understanding of the gradient concept.

Another weakness was on parabolic transformations (question2). Students’ weaknesses were that they did not really understand the values of the coefficients on the quadratic formula. Moreover, they did not even think of drawing the graphs while varying the values of one constant at a time in order to study the pattern.

On the quadratic word problem, a lot of students failed to link and transform the number story to the quadratic functions. This exhibited an APOS level which is far below object level. The moment they saw the quadratic equation, they only thought of solving it without clear understanding of the question. However, most respondents managed to solve it correctly showing that they were able to carry the actions correctly implying that they fall in the action level of APOS.

5.3 Recommendations
5.3.1 Affording pre-service teachers more time on Modules containing content they would teach in schools

The fact that the study revealed that pre-service teachers operate at different levels of understanding on concepts, it therefore calls for the need to cater for these human differences by allocating more time to more challenging concepts which are taught in schools. It is also crucial for pre-service teachers to acknowledge that as pupils come into classrooms, they have different levels of grasping concepts. Consequently, it is recommended that first year students be given time for thorough exposure on quadratic function concept which they would teach at ‘O’ Level. This promotes a good understanding of the concept. Most of them have no conceptual understanding of parabolas, vertices, quadratic word problems and quadratic transformations. Some mental constructions may be pushed to a higher level as they interact more with some content. Action level maybe interiorized into process level and the process level may be encapsulated into an object level. They may result in the teachers having problems with selecting suitable methodologies which fosters conceptual understanding of concepts.

5.3.2 Use instructional methodologies which foster in class

The employment of ACE in this research was a fruitful progression because the researcher noticed that as the student teachers interacted with each other in the classroom discussions, slow learners benefited a lot from fast learners. The discussions helped some students to interiorize their mental constructions from action level to process and those at process level to go through encapsulation in order to reach object level. Lecturers are therefore recommended design instructional methodologies that help students to improve their level of understanding of quadratic function concept.

5.3.3 Joining research organizations

Since the research was based on APOS Theory which was fathered by Dubinsky (1984), it is imperative for educators to join research organizations so that they keep abreast with current
information. This also exposes them to a wide variety of teaching methods which fosters relevant mental structures in order to understand a concept.

5.3.4 Misconceptions held by learners

It was found from the study that learners make a lot of misconceptions as they learn concepts. It is therefore imperative for educators to identify these misconceptions and find ways of dealing with them. Current technologies like graphical calculators, computer software can be effectively used in classrooms to reduce misconceptions held by students. Educators share the above important ideas at organizations like Southern African Association Research of Mathematics, Science, Technology and Engineering. (SAARMSTE), which may include misconceptions held by students on specific concepts and how they can be eradicated.

5.3.5 Relating Mathematics concepts to real world situations

ACE teaching methodology involved activities carried outside the classroom, giving the pre-service teachers the chance to relate Mathematical concepts to the real world. It was observed that most students appreciated the importance of Mathematics, hence boosting their attitude towards the subject. From this background, it can be recommended that lectures in colleges and universities can improve pass rates in schools by in cooperating ACE in their teaching rather than lecturing. This fosters the love of the subject in Mathematics educators, leading to an improved understanding of the concepts.

5.3.6 Effectiveness of Exercises and homework

The exercises and homework given to student teachers proved to have reinforced the learnt concepts taught. Students had the opportunity to build on their mental constructions as they kept on practicing the learnt concepts. Educators are therefore recommended to use homework and exercises effectively as this may push students’ levels of understanding higher.

REFERENCES


**Validity and Reliability**

In qualitative research, “validity might be addressed through the honesty, depth, richness and scope of the data achieved, the participants, the extent of the triangulation and the disinterestedness or objectivity of the researcher” (Winter as cited in Cohen et al. 2007: 133). Validity can be improved through careful sampling, using the appropriate instruments and data analysis techniques (Henning 2004). Validity is not something that can be achieved absolutely but it can be maximized. According to Cohen et al. (2007: 149), reliability can be seen as the correlation
between the researcher’s recorded data and what actually happens in the natural setting of the research. This was achieved by analyzing the pre-service teachers’ written responses against the itemized genetic decompositions formulate

The purpose of the study was to investigate the influence of teacher related factors on students’ performance in KCSE in public secondary schools in Kibwezi Sub County. Four research objectives guided the study. The objectives sought to establish the influence of teacher job satisfaction on students performance in KCSE exam, determine the influence of teacher motivation on students performance, assess the influence of teacher professional qualification on students performance in KCSE examination and lastly to establish the influence of teacher professional experience on students performance in KCSE in Kibwezi Sub County. The study used descriptive survey design and stratified sample was used to select the respondents. A sample of 18 principals, 90 teachers and 180 students was used. The study revealed that teacher job satisfaction influenced students performance as indicated by responses from 17(94.4%) of the principals and 74 (86.0%) of the teachers. It also revealed that teachers motivation is a key factor influencing students performance as indicated by responses from 14 (77.8%) of the principals. The study established that teacher professional training influenced students’ performance because such teachers utilize their acquired skills and talents better. This was agreed upon by 12 (667%) of the principals, 93 (54.7%) of the teachers and 89(52.4%) of the students. Teacher professional experience was also found to have a great influence on students” performance. Experienced teachers found their jobs more enjoyable, meaningful and performed then jobs move effectively. This was indicated by 11 (61.1%) of the principals and 71 (82.6%) of the teachers. The study recommends that schools should motivate their teachers more the ministry of education should provide more opportunities for further professional training and that newly employed teachers should be given on the job training to enhance performance

2.5 Identification of the research gap
The above reviewed literature presents studies carried out in different parts of the globe, on matters pertaining to the teacher factors affecting the performance of learners. These studies have been carried out in other countries, but only a few studies have been carried out in Kenya which are addressing factors affecting students performance such as school environment, instructional materials, efficiency in utilization of the specified teaching period but none has addressed the teacher related factors in Kibwezi Sub County. This study aimed to fill the missing knowledge gap on the teacher factors affecting the performance in Kenya, with the information and data obtained from public secondary schools in Kibwezi Sub County.

26
2.6 Theoretical framework
This study was guided by Affective Events Theory
Appendices
13 April 2016

TO WHOM IT MAY CONCERN

RE: NAME: TENDERE JANG  REGISTRATION NUMBER: B1441325
PROGRAMME: MSc Ed MT  PART: 2.1

This memo serves to confirm that the above is a bona fide student at Bindura University of Science Education in the Faculty of Science Education.

The student has to undertake research and thereafter present a Research Project in partial fulfillment of the Diploma in Science Education/Bachelor of Science Degree/Bachelor of Science Honours Degree/Masters of Science Education Degree programme. The topic of the research is:

AN EXPLORATION OF A MATHEMATICAL CONCEPT USING APOS THEORY AT MUTARE TEACHERS' COLLEGE

In this regard, the department kindly requests your permission to the student to carry out his/her research in your institution.

Your co-operation and assistance is greatly appreciated.

Thank you

[Signature]
C Denhere (Prof.)
Co-ordinator

BINDURA UNIVERSITY OF SCIENCE EDUCATION
DEPARTMENT OF EDUCATIONAL FOUNDATIONS

[Stamp with date: 15 APR 2016]

P Bag 1020
BINDURA, ZIMBABWE
Tel: 0271 – 7531 ext 1038
Fax: 263 – 71 – 7616
Email: cdenhere@buse.ac.zw
APPENDIX B

Letter of Consent for Pre-service teachers

Bindura University of Science Education

P Bag 1020

Bindura Zimbabwe

Dear Mathematics student teacher

Request for your participation in a Research Project

I am a Masters student at the above named University, specializing in Mathematics Education. My research area uses Action-Process-Object-Schema (APOS) theory to explore student teachers’ levels of understanding of quadratic function concept. This requires me to conduct a study with pre-service teachers in order to find out their levels of understanding as they learn the concept. The researcher hereby requests your consent to be a willing participant in the study. It is assured that fictitious names of participants will be used and the information obtained will be kept confidential upon the release of findings of the study. The findings of the study will be used for the purposes of the study objectives only. Your participation may be withdrawn if you so wish. For clarifications, you can contact Tendere Jane (0716696062).

Yours, faithfully

Tendere. J
Read and sign

I --------------- ------------------------------------, student teacher at Mutare Teachers’ College, hereby declare that I fully understand the contents of the above letter and understood that I am have no objection in participating in this study. However, if need arises, I will be free to terminate my participation at any moment. I also understand that real names will not be used in reporting the findings and that the names will always be kept confidential.

Signature: ------------------------

Date: --------------------------
Appendix C

Questionnaire for Mathematics Lecturers

I am a student with the Bindura University of Science Education (BUSE) reading for the MEDSC (Mathematics) degree. I am carrying a research on evaluation of quadratic functions using Action Process Object and Schema (APOS) theory. The information you are going to give will be used only for this study and shall be treated with confidentiality. Your honest contribution will be appreciated. Please do not write your name on the questionnaire.

Part A: Demographic information.

Please respond to the items by putting a tick (✓) on the most appropriate column for you.

<table>
<thead>
<tr>
<th>Experience in teaching first years: Sets, functions and Coordinate Geometry</th>
<th>None</th>
<th>At least once</th>
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<tbody>
<tr>
<td>Response</td>
<td></td>
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<table>
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<tr>
<th>Lecturing Experience</th>
<th>9 &amp; less yrs</th>
<th>10-14 yrs</th>
<th>15+ yrs</th>
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<tbody>
<tr>
<td>Response</td>
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<tr>
<th>Qualification</th>
<th>1st degree</th>
<th>2nd degree</th>
<th>3rd degree &amp; further</th>
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<tbody>
<tr>
<td>Response</td>
<td></td>
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<table>
<thead>
<tr>
<th>Type of Qualification</th>
<th>CE/Diploma in Education</th>
<th>Degree in Education</th>
<th>No degree in Education</th>
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<tbody>
<tr>
<td>Response</td>
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Part B

Please indicate by a tick (✓) the most appropriate response from the following:

1. Strongly disagree (SD)
2. Disagree (D)
3. Uncertain(U)
4. Agree (A)
5. Strongly Agree (SA)

Give reason(s) for your response in the space provided to the right of your response.

<table>
<thead>
<tr>
<th>Understanding of Quadratic functions</th>
<th>SD</th>
<th>D</th>
<th>U</th>
<th>A</th>
<th>SA</th>
<th>Reason(s) for your response</th>
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<tbody>
<tr>
<td>1) Students’ understanding of quadratic functions is procedural rather than conceptual.</td>
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<td>2) We teach students to acquire conceptual understanding of concepts at this college.</td>
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<td>3) There are different ways of representing quadratic functions which students are very familiar with.</td>
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<td>4) Some theories can be used to evaluate student’ understanding of the quadratic functions.</td>
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<td>..................................................</td>
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<td>5) Students’ understanding of concepts depends on a variety of factors.</td>
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<table>
<thead>
<tr>
<th>Applicability of quadratic functions to everyday situations</th>
<th>SD</th>
<th>D</th>
<th>U</th>
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<td>6)</td>
<td>Students clearly understand that a quadratic function is graphically represented by a parabola.</td>
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<tr>
<td>7)</td>
<td>Characteristics of parabolic graphs are clear to students (intercepts, symmetry, vertices, focus directrix, e, t, c)</td>
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<td>8)</td>
<td>Students can relate parabolic functions to real life situations.</td>
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<td>9)</td>
<td>When given problems on maximum/minimum heights or revenues, they can clearly relate to vertices.</td>
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<tr>
<td>10)</td>
<td>Understanding of parabolic functions is important in everyday situations.</td>
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<td>11)</td>
<td>The concept; quadratic functions is linked to most subjects in the secondary school curricula.</td>
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<tr>
<td>12)</td>
<td>Everyday problems can be transformed to parabolic functions.</td>
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<td>13)</td>
<td>Students require certain mental structures in order to understand parabolas.</td>
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<td>14)</td>
<td>Attitude is also related to the ability to understand concepts.</td>
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<td>15)</td>
<td>Students are interested in concepts which can be graphically represented, like parabolic functions</td>
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</tbody>
</table>

**Teaching Approaches**

<table>
<thead>
<tr>
<th>Teaching Approaches</th>
<th>SD</th>
<th>D</th>
<th>U</th>
<th>A</th>
<th>SA</th>
</tr>
</thead>
<tbody>
<tr>
<td>16) Some lecturers do make lectures learner oriented.</td>
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<tr>
<td>17) There is enough time allocated to teach parabolic functions on the college timetable.</td>
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<tr>
<td>18) There is need to change the teaching approaches in order to improve on the conceptual understanding of concepts</td>
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<td>19) There is a relationship between teachers’ understanding of a concept and the way he/she teaches.</td>
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<td>20) Learner centred approaches may enhance understanding of concepts taught.</td>
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</table>

THANK YOU
Appendix D

Test for pre-service teachers

Instructions

- Answer all questions
- Show clear working
- Write neatly.

1) a) Define a quadratic function. (2)
   b) Explain how you would introduce this concept to a ZJC class. (4)

2) Discuss the relation of a; b; c in the quadratic function \( ax^2 + bx + c \) (standard form of a Quadratic function). What do these coefficients do the graph vertex? (3)

3) Consider the following questions for the function \( f(x) = x^2 + 4x + 4 \).
   a) Draw the graph (1)
   b) The vertex is (---, ---) and is a minimum or maximum? (2)
   c) Write the quadratic function in vertex form. (3)
   d) How would you explain the vertex concept in relation to axis of symmetry to a ZJC class? (2)

4) The height \( h \), in metres of an object above the ground is given by \( h = 16t^2 - 64t + 19 \), where \( t \geq 0.5 \), where \( t \) is time in seconds. Find the time it takes the object to strike the ground and find the minimum/maximum height of the object (3)

Total 20 marks
Appendix E

The questionnaire consisted of twenty questions which were subdivided into three sections: understanding of quadratic functions (items 1-5), applicability of quadratic functions to everyday situations (items 6-15) and teaching approaches (items 16-20).

The table below deals with understanding of quadratic functions (items 1-5)

<table>
<thead>
<tr>
<th>Item</th>
<th>Statement</th>
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</thead>
<tbody>
<tr>
<td>1</td>
<td>Students’ understanding of quadratic functions is procedural rather than conceptual.</td>
</tr>
<tr>
<td>2</td>
<td>We teach students to acquire conceptual understanding of concepts at this college.</td>
</tr>
<tr>
<td>3</td>
<td>There are different ways of representing quadratic functions which students are very familiar with.</td>
</tr>
<tr>
<td>4</td>
<td>Some theories can be used to evaluate student’s understanding of the quadratic functions.</td>
</tr>
<tr>
<td>5</td>
<td>Students’ understanding of concepts depends on a variety of factors.</td>
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</tbody>
</table>
Appendix F

Data obtained from lecturers’ questionnaire on pre-service teachers’ understanding of quadratic function concept

<table>
<thead>
<tr>
<th>Lecturer</th>
<th>Item 1</th>
<th>Item 2</th>
<th>Item 3</th>
<th>Item 4</th>
<th>Item 5</th>
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</thead>
<tbody>
<tr>
<td>A</td>
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<td>4</td>
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<td>B</td>
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<tr>
<td>C</td>
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<td>3</td>
<td>2</td>
<td>4</td>
<td>5</td>
</tr>
</tbody>
</table>

Key

- Strongly Agree: 5
- Agree: 4
- Undecided: 3
- Disagree: 2
- Strongly Disagree: 1
Appendix E

Lecturers’ responses on items 16 -20

<table>
<thead>
<tr>
<th>Lecturers</th>
<th>Item16</th>
<th>Item17</th>
<th>Item18</th>
<th>Item19</th>
<th>Item20</th>
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<tr>
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</tr>
</tbody>
</table>

Key

- Strongly Agree 5
- Agree 4
- Undecided 3
- Disagree 2
- Strongly Disagree 1