COGNITIVE OBSTACLES FACED BY ADVANCED LEVEL MATHEMATICS STUDENTS WHEN LEARNING PARTIAL FRACTIONS: A CASE STUDY OF A SCHOOL IN MANICALAND

BY

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Abstract

The purpose of this study was to assess the cognitive obstacles faced by Advanced level students when learning partial fractions. A case study of a school in Manicaland province. Three basic research questions were constructed, which emphasized the extent to which students cognitive obstacles hindered the students’ learning and the specific obstacles.

Vygotsky theory of concepts development was used as an appropriate theoretical framework to use to capture the ‘A’ level students’ cognitive obstacle when solving learning partial. The procedures employed included classteaching, test and the interviews administered to the students in order to find out the obstacles. To conduct this study, purposive sampling technique was used. Among the 25 students doing Advanced level at the school 15 of them were selected using purposive sampling. From these sample of students, 8 were girls and 7 were boys were all used for purposive sampling technique. Interviews were used as main tool of data collection. The test was also used to substantiate the data gathered through interviews. Tables were used analyse the marks got from the test. The flow chart was also used to analyse the data obtained from the interviews. The qualitative data gathered was analysed by narration. The results of the study showed that students used their intuitive knowledge, inability to use the correct mathematics and imperfect mathematical knowledge. On the other angle their are other factors such as lack of comprehension and use of inappropriate strategies also causing obstacles.
Declaration

The researcher hereby declares that the thesis on the title, Cognitive obstacles faced by ‘A’ level mathematics students when learning partial fractions: A case study of a school in Manicaland province, is his original work and that all sources that have been referred to and quoted have been indicated and acknowledged with complete references.

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Table of Contents

Table of contents

Contents

Abstract .............................................................................................................................................. i
Declaration ......................................................................................................................................... iii
Acknowledgments ............................................................................................................................ iv
CHAPTER 1 ........................................................................................................................................ 1
   Introduction .................................................................................................................................... 1
   1.1 Background to the study ........................................................................................................... 1
   1.2 Statement of the problem ......................................................................................................... 3
   1.3 Research question ................................................................................................................... 3
       1.3.1 Sub questions ................................................................................................................... 3
   1.4 Assumptions of the study ......................................................................................................... 4
       1.4.1 Theoretical framework ..................................................................................................... 9
   1.5 Significance of the study ......................................................................................................... 4
   1.9 Summary ................................................................................................................................... 6

CHAPTER 2 ....................................................................................................................................... 7
   2.0 REVIEW OF RELATED LITERATURE .................................................................................... 8
       2.1 Introduction ........................................................................................................................... 8
       2.1.1 Epistemological and pedagogical obstacles faced by students when learning algebra .... 11
       2.1.2 DIDACTICAL OSTACLES ............................................................................................. 20
       2.2 Summary ............................................................................................................................. 21

CHAPTER 3 ....................................................................................................................................... 21
   3.0 RESEARCH METHODOLOGY ................................................................................................. 21
3.1 Introduction ......................................................................................................................... 21
3.2 Research Design. ................................................................................................................. 22
3.3 Instruments used to collect data ......................................................................................... 23
3.3.2. Interview Guide ............................................................................................................. 24
3.3.3 Interview ......................................................................................................................... 25
3.4. Population, sample and sampling techniques to be used in the study ......................... 25
3.5 Procedures of collecting data ............................................................................................ 25
3.5.1. INTERVIEW GUIDE ................................................................................................. 26
3.5.2. Step 1: Class teaching ................................................................................................. 27
3.5.4 Step 3: Interviews .......................................................................................................... 27
3.6 Data presentation and analysis procedures ........................................................................ 29
CHAPTER 4 .............................................................................................................................. 29
4.0 DATA PRESENTATION, ANALYSIS AND DISCUSSION ................................................. 29
4.1. Introduction ....................................................................................................................... 29
4.2. Data presentation ................................................................................................................ 30
4.2.1. Analysis of Interview Data......................................................................................... 30
4.3.RESULTS AND DISCUSSION ......................................................................................... 31
4.3.1. Lack of comprehension of the question asked............................................................ 34
4.3.3. Inability to translate the problem into a mathematical form involving partial fractions .... 35
4.3.4. Failure to apply correct mathematics ......................................................................... 35
4.3.5. Inappropriate strategy used on operations .................................................................... 36
The following analysis shows epistemological and didactical obstacles obtained from research. 38
4.3.7. Imperfect mathematical knowledge ............................................................................ 38
4.3.8. Misinterpretation of the problem ................................................................................ 38
4.3.10. Table 8: Actual marks of students for test. ............................................................... 39
CHAPTER 5 .......................................................................................................................... 42
5.1. Introduction ................................................................................................................. 42
5.2. Summary of the project including constraints ......................................................... 42
5.3. Conclusions .................................................................................................................. 43
5.4. Recommendations ....................................................................................................... 45
List of tables page

Table 1: Interview.................................................................................................................26
Table 2: Structure derived from Newman (1983) and Ransely (1979).................................28
Table 3: Categorisations in behavioral terms used for interview analysis............................30
Table 4: Flowchart showing some of the students’ obstacles..............................................33
Table 5: Analysis of obstacles from the flow chart...............................................................33
Table 6: Pedagogical obstacles..............................................................................................37
Table 7: Grouped Students’ marks.........................................................................................39
Table 8: Students’ actual marks in the test...........................................................................39
CHAPTER ONE

Introduction
This chapter deals with background of the study, statement of the problem, research question of the study, significance of the study, delimitation of the study, limitation of the study and definition of operational terms used in the research

1.1 Background to the study

According to Newman (1983), obstacles in problem solving may occur at one of the following phases, namely reading, comprehension, strategy know-how, transformation, process skill and solution. Schoenfeld (1985) suggested four aspects that contributed to problem-solving performance which are the problem solver’s mathematical knowledge, knowledge of heuristics, affective factors which affect the way the problem solver views problem solving and managerial skills connected with selecting and carrying out appropriate strategies.

In their study of the problem-solving research literature, Kroll and Miller (1993) identified three major cognitive and affective factors; namely, knowledge, control (metacognition) and beliefs and affects that contributed to students’ obstacles in problem solving. Further Lester (1994) expressed that obstacles experienced during problem solving could also be caused by the problem solver’s characteristics such as spatial visualization ability to attend to the structural features of problems, dispositions such as beliefs and attitudes and experiential background such as instructional history and familiarity with types of problems.

In the early 1970s, research tended to attribute obstacles in solving problems to the various tasks variables such as content and context variables, structure variables, syntax variables and heuristic
behaviour variables (Goldin & McClintoch, 1979). However, Lester (1994) contended that there was a general agreement that problem difficulty is not so much a function of various task variables but rather a function of characteristics of the problem solver. In other words, the knowledge one possesses, one’s disposition and one’s experiential background often influence problem solving performance.

Advanced level students face obstacles when learning partial fractions. The obstacles are indicated by the errors and mistakes that are made when they are solving problems involving partial fractions (Zimsec 2010). These problems occur every year when they write public examinations and during in class tests. Therefore the researcher has been motivated to investigate the causes of the cognitive obstacles when learning partial fractions. The cognitive obstacles cause students to develop misconceptions as they learn partial fractions. The researcher is mainly interested in the cognitive obstacles encountered because students make errors and avoid answering questions on partial fractions in tests and examinations.

Mathematics workshops and meetings carried out at cluster and district level have shown that teachers must be staff developed and try to find ways to teach partial fractions. A lot of arithmetic and algebra involved in learning partial fractions also cause students to make mistakes as they carry out computations.

Problem solving in mathematics is a process which calls for the student to be engaged with skills and knowledge to solve the problems. To successfully solve various types of problems, in particular the non-routine ones, a student has to apply four types of mathematical facilities, namely, specific mathematics concepts, skills, processes, and metacognition to tackle the problem. Mandler (1989) therefore went on to emphasise the importance of analysing the problem task and learnerto
ensure that surprises, errors and mis-steps can be handled by some alternatives routes, substitute actions, or a rewording of the task.

1.2 Statement of the problem

Students face cognitive obstacles when learning partial fractions at Advanced level mathematics. It is known, locally and internationally, that students learn algebra with great difficulty. These difficulties cause the students to develop obstacles when learning partial fractions. Poor students’ performance in science subjects in Secondary Schools such as Mathematics and Sciences is an issue that has been well known and discussed by many people for so long in Zimbabwe (Mtetwa, 2000). Many students fail mathematics in public examinations and a few students are taking the subject up to Advanced level. Some of the obstacles arise as a result of students’ previous experiences.

1.3 Research question

What are the cognitive obstacles faced by students when learning partial fractions? How are these obstacles manifesting in students.

1.3.1 Sub questions

(a) What are the epistemological obstacles faced by students when solving partial fractions

(b) What pedagogical obstacles that are faced by students when learning partial fractions.

(c) What are the didactical obstacles faced by students when learning partial fractions.
1.4 Assumptions of the study

In carrying out the study it is assumed that the teachers who teach mathematics are qualified and well experienced. The teachers can teach students using different approaches and are able to adjust to the students’ special needs. The teachers also take into account the differences between gifted and ungifted learners when pacing their lessons. Furthermore, the teachers who are teaching mathematics at Advanced level are committed to their work and are highly motivated and the students have been streamed. The students have passed their Ordinary Level mathematics and have clear understanding of concepts. The students have learnt elementary algebra including how to add, subtract, divide and multiply. However besides the above assumptions students are facing obstacles when learning partial fraction decomposition.

1.5 Significance of the study

The purpose of the study on cognitive obstacles is to improve instruction, and helping teachers ensure effective teaching and be aware of the students’ difficulties. Teachers can benefit on the way they deliver their lessons and improve methodologies they use when teaching the students because they will be aware of students’ difficulties. If students are sensitized on areas they make mistakes and errors they will find means and ways of trying to minimize the errors.

Educational officers can also benefit, by knowing the obstacles faced by students in the schools, they can use the information to discuss at cluster and district level the problems affecting the students, hence workshops can be arranged to present this information as a way of staff developing
teachers. The study can help, the Curriculum Development Unit, on how to construct their syllabi and the way topics can be sequenced so that students can make a link on the topics.

1.6 Limitations of the study

Limitations are those conditions beyond the control of the researcher that may place restrictions on the conclusions of the study, (Tuchman, 1978:58). These are weaknesses that are inherent in the research and should be taken note of. There is a limitation on generalizability of the findings which the researcher obtained because this research was restricted to one school.

Absenteeism of pupils from lessons due to different reasons compromised the objectivity of the research. However, to reduce this dilemma the researcher had to frequently organize with the school administration. Moreover the research design to be used have its own loopholes and fail to capture other crucial information as the results cannot compare with other schools. The questioning technique to obtain answers might not be right to obtain all the answers hence they should be designed in a good way.

The students might have been taught the topic on partial fractions in short time for example, two weeks instead of the normal maximum days required hence the students will perform badly in the given tests or examinations. If less time was advocated for the students, they definitely perform badly.
1.7 Delimitations of the study

According to the Oxford Advanced Dictionary (1963) delimitation refers to the limits or boundaries of the study. Simon (2011) argues that delimitations are those characteristics that limit the scope and define the boundaries of the study. Simon went on to state that delimiting factors include choice of objectives, the research questions, variables of interest, theoretical perspectives adopted and the population the researcher decides to investigate. This also includes the criteria of participants to enrol in the study, the geographic region of the study and the organization involved. The validity of the research is confined to the school under study only and not generalized to other schools in the district or the province. The study has been confined to what is current in the school understudy.

1.8 Definition of Key words

Obstacles are barriers or impediments that block or prevent the progress or achievement of concrete goals.

Cognitive is the mental action or process of acquiring knowledge and understanding through experience.

Partial fractions refers to two or more fractions into which a more complex fraction can be decomposed as a sum.
1.9 Summary

Research has indicated that students often meet obstacles as they learn mathematics at Advanced level. However these obstacles are caused by students’ mental structures as they try to bring in new information in the already existing one to solve problems. According to Vygotsky students meet different complexes as they try to solve problems as result these complexes can cause students to learn new concepts properly. However these complexes should help the student to develop knowledge.
CHAPTER TWO

2.0 REVIEW OF RELATED LITERATURE
2.1 Introduction

Literature states that students face cognitive obstacles when learning partial fractions. The notion of cognitive obstacles was first introduced in the realms of science by Bachelard (1938) and highlighted in mathematics education by Brousseau (1984). In their terms an obstacle is a piece of knowledge of the student that has in general been satisfactory for a time for solving certain problems and so becomes anchored in the mind, but subsequently, when faced with new problems, it proves to be inadequate and difficult to adapt (Bachelard 1938).

Brousseau classifies cognitive obstacles as ontogenetic, didactical and epistemological. Herscovics (1989) used the term cognitive obstacle to refer to either the existing mental structure (or its attempted use) or the structure of the new material. According to Herscovics (1989) he believed that learning difficulties were of two basic types:

1. The learner attempts to map new material onto an existing mental structure which is valid in another domain but inappropriate for the knowledge to be learned.

2. The inherent structure of the new material might be such that the learner has no existing mental structure which would allow assimilation of the new material. Kinds of cognitive obstacles stated by Herscovics (1989) identified from the work of Bachelard (1938) include, the tendency to rely on deceptive intuitive experiences, the tendency to generalize and obstacles caused by natural language. On the other hand Cornu (1991) differentiates between four types of obstacles: cognitive obstacles, genetic and psychological obstacles, didactical obstacles, and epistemological obstacles.
According to Cornu, cognitive obstacles are a product of the student’s previous experience and their internal processing of these experiences and are manifested when students encounter difficulties in the learning process. According to Cornu (1991) genetic and psychological obstacles also referred to as ontogenic obstacles occur as a result of personal development of the student. Didactical obstacles are caused by instructional choices and therefore, are avoidable through the development of other instructional approaches. Epistemological obstacle, arise regardless of the instructional approach, for they arise in the nature of the mathematical concepts themselves. These descriptions by Cornu seem to give an impression that there is a clear distinction between these kinds of obstacles. However, Tall (1989) used the term ‘cognitive obstacle’ for epistemological obstacles.

2.2. Theoretical framework

Vygotsky(1986) spoke of three types of pre-conceptual thinking, heaps, complexes and potential concepts each of which roughly corresponds to a different stage of the development of generalization and abstraction in the individual. Vygotsky’s theory of concept formation (1986) is a powerful framework within which to explore how an individual at Advanced level constructs a new mathematical concept. Vygotsky characterised complex thinking in five different ways: the associative complex, the chain complex, the collection complex, the diffuse complex and the pseudoconcept.

With heap thinking, the person links ideas or objects together as a result of an idiosyncratic association. This form of thinking (also called thinking in ‘syncretic images’) is an initial stage in the child’s development.
With complex thinking, ideas are based on experience and associations rather than on logic or a system but the learner is able to abstract actual attributes of the idea. In Vygotsky’s theory, complex thinking involves the development of generalisations upon which further and more refined generalisations and abstractions may be based. However in these generalisations obstacles can be encountered.

For associative complex, unfamiliar objects are artificially associated with a familiar object: When confronting a new mathematical object or expression, part of the object or expression (perhaps a sign or symbol) may remind learners of another mathematical sign with which they are more familiar and which is epistemologically more accessible. This more familiar sign may then become the nucleus of the new concept. But the connection between the two signs may be artificial or not relevant. For example, students often associate the arithmetic, closure to the statement $\frac{1}{6} + \frac{1}{3}$ is response of $\frac{1}{2}$, while in algebra, the statement $\frac{x}{6} + \frac{1}{3}$ is a final answer by itself. However students would add to give $\frac{x+1}{2}$ hence an obstacle has been encountered. The distributive law to simplify $\frac{x^2+3}{2x-1} + \frac{x}{2} = \frac{2(x^2+3)+x(2x-1)}{2(2x-1)}$ but the students expand $2(x^2 + 3) + x(2x - 1)$ as $(2x^2+3)+2x^2-1$ which means their is difficulty to recognise the distributive law. There is also problem in the decomposition of a rational functions into partial fractions, due to failure to match into the right form to be used.

For example $\frac{x}{(x^2+4)^3} = \frac{Ax+B}{x^2+4}$ forgetting that it is involving repeated factors.

In the chain complex, an object is included in a group because it shares an attribute with another object already in the group. The new object enters the group with all its attributes and the learner then uses any of these attributes to include yet another object with a shared attribute in the group.
With conceptual thinking, the bonds between the parts of an idea and between different ideas are logical and the ideas form part of a socially-accepted system of hierarchical knowledge. For example, a person who is able to classify an animal as a fish according to specific and systematic attributes contained in its scientific definition is using conceptual thinking

2.3. Epistemological and pedagogical obstacles faced by students when learning algebra

Most of the earlier and well established research on children's understanding of algebra has been influenced by a Piagetian perspective. Herscovics (1989) carried out research in algebra on why pupils do not perform well on certain algebraic tasks. Herscovics discussed some of the research results on the teaching and learning of algebra focusing on the idea of a cognitive obstacles. Herscovics (1989) stated that from the Piagetian perspective, the acquisition of knowledge is a process involving a constant interaction between the learning subject and his or her environment. This process of equilibration involves not only assimilation of the things to be known into some existing cognitive structure but also accommodation changes in the learner's cognitive structure necessitated by the acquisition of new knowledge. However, the learner’s existing cognitive structures are difficult to change significantly, their very existence becoming cognitive obstacles in the construction of new structures. This is where epistemological obstacles come into play, which is knowledge possessed by the student for some time to solve problems but fails to work when facing new problems. This is caused by the difficult inability to change the existing cognitive structure.
Herskovits (1989) in his research on “Cognitive obstacles when learning Algebra” discovered that algebra pupils view algebra expressions such as \((x+7)\) as incomplete. Collins (1980) explains this as being due to the pupils’ inability to hold unevaluated operations in suspension, suggesting that it is not until the stage of formal operations that the pupil breaks away from the idea that algebraic expressions give unique results. Herscovics stated that students have cognitive difficulty accepting a procedural operation as part of an answer. That is, in arithmetic, closure to the statement \(\frac{1}{6} + \frac{1}{3}\) is a response of \(\frac{1}{2}\) while in algebra, the statement \(\frac{x}{6} + \frac{1}{3}\) is a final entity by itself (Booth, 1988; Davis, 1975). Student errors in using algebra algorithms often are not due to failing to learn a particular idea but from learning or constructing the wrong idea (Matz, 1980). Lincheski and Herscovics (1994), conducted a study that found Grade six students over-generalised order of operations, failed to perceive cancellation of terms in an equation as they operated, algebraic skills. Therefore, poor understanding of algebra at elementary level causes a manifestation of epistemology. When learning partial fractions which is also part of algebra, epistemological obstacles are also encountered when decomposing rational functions into partial fractions, as students fail to solve for the value of constant \(A\) and constant \(B\) as shown below when expressing \(\frac{5x+7}{(x-1)(x+3)}\) in partial fraction of the form \(\frac{5x+7}{(x-1)(x+3)} = \frac{A}{x-1} + \frac{B}{x+3}\) so when solving for the value of \(A\) and \(B\) students write it as \(\frac{5x+7}{(x-1)(x+3)} = \frac{A(x+3)+B(x-1)}{(x-1)(x+3)}\) students have find the overall common denominator instead of multiplying both sides by \((x-1)(x+3)\) so that like terms cancel and equate powers of equal degrees to find \(A\) and \(B\).

Carpenter et al (1989), the National Assessment for Education in the United States of America, investigated, how the solution of first degree equations in one unknown prior to instruction is solved by students. According to Herscovics (1989) there exist a cognitive gap on arithmetic and
algebra, which is characterised as the students’ inability to operate spontaneously with or on the unknown was being faced by the students. Students fail to detach a numeral from the preceding minus sign in the grouping of numerical terms and problems in the acceptance of the equal symbol to denote a decomposition into a difference as in $23 = 37 - n$ which leads some students to read such equations from right to left. There is difficulty experienced due to the transition from equations to identities by using the symbol ($=, \equiv$). When using the equal sign, the equation hold for a single value of $x$. However, for an identity, it holds for any value of $x$. For example find the values of $A$, $B$ and $C$, in $(5x + 7 \equiv A(x + 3) + B(x - 1))$. Students confuse the equal sign and identity symbol.

The research also indicated that, students often face challenges when learning algebra. Thus, only about half the students emerge from a first course. Carpenter et al. (1981) reported that 67% of the 17-year-olds in their sample had completed a first course in algebra and that only 35% had taken at least a half year of a second course. Eight years later, Swafford and Brown (1989) reported that for the same age group, 75% had completed first-year algebra while 40% had completed a second year. Thus, only about half the students emerge from a first course in algebra, traditionally involving one unknown, with sufficient motivation to enroll in a course dealing with algebra in two variables. This shows that when it comes to partial fractions involving two variables $A$ and $B$ which are constants to be calculated students often also face obstacles.

Cognitive obstacles arise in the transition between arithmetic and algebra limiting the students in the way they learn. Davis’ (1975), carried out a clinical interview of an exceptionally bright 12-year-old student (Henry), enrolled in an experimental class following an enriched algebra program
(Davis, 1975). After being taught how to solve equations such as \(3x + 2 - 5x + 6 = 9z - 3x + 23\), the class was introduced to rational forms such as \(\frac{3}{x} = \frac{6}{3x+1}\) upon being told to multiply both sides by \(x\), Henry seemed to agree that by multiplying \(\frac{3}{x}\) by \(x\) he would be left with 3. However, when told to multiply the right-hand side by \(x\), he replied: "How can we multiply by \(x\) when we don't know what \(x\) is?" This kind of evidence is worth mentioning for it so clearly indicates the cognitive problem at hand. Although Henry may view the literal symbol as a generalized number, he cannot operate with it. This problem can also arise in the learning of partial fractions, for example when solving \(\frac{2x}{(x+3)(3x+1)} = \frac{A}{x+3} + \frac{B}{3x+1}\) for the values of \(A\) and \(B\), Students multiply the left side by \((x + 3)(3x + 1)\) and left with \(2x\), however on the right side they just multiply the denominators by \((x + 3)(3x + 1)\) giving the equation as \(2x = \frac{A}{(x+3)^2} + \frac{B}{(x+1)^2}\). This shows an epistemological obstacle, since the student is failing to cancel terms which are similar in the cross multiplication of the numerators, instead student multiplies the denominators.

More substantial evidence can be gathered from a large scale assessment study carried out in Great Britain involving 3000 secondary school students in their second, third or fourth year aged 13.3, 14.3, and 15.3 respectively (Kuchemann, 1981, 1978). Only results obtained from the 14-year-olds in their second year of algebra have been published. When asked to add \(\frac{4}{x}\) onto \(\frac{3n}{2x}\), 36% gave a correct answer \(\frac{3n+8}{2x}\), but 31% answered \(\frac{4+3n}{3x}\) while 16% gave \(\frac{7}{3x}\) as the answer. When asked to add 4 onto 3n, 36% gave a correct answer 3n + 4, but 31% answered 7n while 16% gave 7 as the answer. When asked to multiply \(n + 5\) by 4, 17% gave correct answers \(4n + 20\) or \(4(n + 5)\), while 19% answered \(4n + 5\) or \(4(n + 5)\), 31% gave \(n + 20\) as the answer, and 15% simply
wrote 20. The last two results indicate that at least 46% of the students will not perform the required operation on the algebraic symbol. Of course, some of the difficulties faced by the British students are specific to algebraic expressions. One cognitive problem identified with this mathematical form is what Davis (1975, p. 18) called the "name-process” dilemma by which an expression such as 6z is both an indication of a process ("What you get when you multiply 6 by z") and a "name for the answer". Sfard and Linchevski (1993) have suggested that the term "process-product dilemma" better describes this cognitive problem. Therefore when learning partial fractions, students also face the problem of failing to perform the distributive law properly when multiplying algebraic expressions. For example, solving \( A(y + 4) \), some students multiply A by \((y + 4)\), meaning the student has associated \( A + (y + 4) \) with \( A(y + 4) \).

Looking on the other angle when students are asked to simplify or express a certain type of partial fraction as a single fraction, some students have a tendency, to fail performing required operation on the algebraic symbols. Failure to see or recognise whether there is an equal sign or not as an illustration, \( \frac{13}{2x+7} + \frac{6x}{x+1} \) is written as \( 13(x + 1) + 6x(2x + 7) \). This means that the student is trying to solve \( \frac{13}{2x+7} + \frac{6x}{x+1} \) as an equation instead of simplifying it into a single fraction. All these kinds of obstacles cause students to perform poorly, cause a lot of hindrances in their learning.

Research in cognitive science is increasingly challenged by educational studies into how students learn subject matter and as a result has revealed the importance of knowing the cognitive obstacles engaged in during learning. In learning algebra knowledge is a hierarchical network of concepts and attributes, connected by relational propositions which are organised from simple to complex concepts (Ericsson & Polson, 1988). It is a cumulative and gradual development with quantitative
and qualitative changes occurring in both what is learnt and how learning occurs (Gott, Kane, & Lesgold, 1994; Halford & Boulton-Lewis, 1992). In the process of learning there is a relationship between algebra and arithmetic.

In his research entitled ‘Understanding Algebra’, Kieran (1992), distinguished between procedural (such as solving $2x + 5 = 11$) and structural (such as simplifying $3x + y + 8x$) conceptions of algebra. She implies that these conceptions are hierarchical, with the procedural conception being at the lower level. Thus students need to form increasingly abstract views of arithmetic; for example, they need to view addition in algebra as an object. So, students face obstacles in the learning of partial fractions which is part of algebra because the way they apply the solving techniques is not systematic and gradual application of concepts. There is no hierarchical application of concepts by students, when solving problems.

A sound arithmetic knowledge base has been recognised as essential to operating within an algebraic framework. For example, Booth (1989) stressed the importance of students' understanding various structural notions in arithmetic. Herscovics and Linchevski (1994) exemplified the importance of a good grounding in arithmetic by analysing the knowledge required to solve $4 + n - 2 + 5 = 11 + 3 - 5$. They stated that students need to be able to use commutativity to obtain $(n + 4) - 2 + 5$ and then associativity to perform $(4 - 2) + 5$. Booth (1988) reported that students hold an inadequate conception of commutativity believing that division, like addition, is commutative. When writing partial fractions failure to recognise division is not commutative as addition cause students to write $\frac{3x}{2x-6} + \frac{7}{x-2}$ as $\frac{2x-6}{3x} + \frac{x-2}{7}$ which is correct.
Linchevski and Herscovics (1994) conducted a study that found Grade six students over-
generalised order of operations, failed to perceive cancellation of terms in an equation as they
operated sequentially from left to right, and displayed a static view of the use of brackets. Understanding of the distributive law is also essential for algebraic functioning (Demana & Leitzel, 1988). Linchevski and Herscovics (1996) consider that appropriate preparation in
arithmetic in upper primary school can help to overcome obstacles and foster the development of
new pre-algebraic skills. Students overgeneralize while simplifying partial fractions by failure to
use the distributive property causing students generate false statements.

Hitendra Pillay, Lynn Wilss, and Gillian Boulton-Lewis from the University of Queensland
University of Technology in United States of America carried out research on Grade seven and
grade 8 students. Interviews were conducted with Grade 7 students before any formal
algebraic instruction occurred; in Grade 8 after instruction in operational laws, use of brackets, and
solution of arithmetic word and number problems; and in Grade 9 after instruction in finding an
"unknown" in an equation and solving an equation using balance procedures. Questions
investigated were the commutative law on (+, -, X, + in 35? 76=76 ? 35) and distributive
(6 x 13 = 60 + 18) laws; meaning of equals in an incomplete (28 + 7 + 20 = ?) and complete
(28 + 7 + 20 = 60 − 36) equation; meaning of unknown (0 + 5 = 9; x + 7 = 16) and
variable (0 + 5; 3x); and solution of equations using numerical and algebraic processes (3(\(x + 7\)) = 24; \(x + 3 = 2x − 1\)). Individual interviews were videotaped. Students were encouraged to complete each task; if they could not, the interviewer proceeded to the next task.
Results of this study indicated that most students in Grades 7 and 8 did not have a satisfactory understanding of commutative and distributive laws to use as a basis for algebra. Results also showed that most students believed equals, in the incomplete equation, denoted the answer. This perception resulted in inappropriate responses for the commutative law for some students who focused on the equals sign and stated that none of the signs would fit. Thus students not only failed to see the full structure of the equation, they also failed to see the relationship between elements of the problem; and as Scandura (1971) argued, algebra is based on relationships. These results also correspond with the beliefs of MacGregor (1996) and Demana and Leitzel (1988) in that students have inadequate conceptions of these arithmetic principles. It was not until Grade 9 that most students had sufficient understanding of commutative and distributive laws to apply these to linear equations. Such inadequacies point to the need for explicit instruction in these arithmetic principles if cognitive difficulties for students beginning algebra are to be reduced.

For equals in the complete equation, understanding moved from inappropriate in Grade 7, to inappropriate or algebraic in Grade 8, to algebraic (denoting an equal or balanced relationship) in Grade 9.

Kieran (1981) noted that students require an equivalence understanding of equals to operate algebraically. In each grade almost one third of the students interpreted "=" arithmetically, that is each side of the equals sign as the same value. This suggests that while students' knowledge of "=" had developed, there was still a substantial number of students who did not understand "=" in an algebraic sense. Therefore these students would not be able to carry out transformations or, as Linchevski (1995) suggested, understand multidirectional relationships. We believe that providing explicit instruction of equals initially at an arithmetic level then subsequently at a pre-algebraic
level (that is, that each side of equals then each side of the equation is the same therefore operating can occur from either side) will provide the foundation to facilitate movement from an arithmetic to algebraic understanding of equals.

Most students, over the three years, knew that D in the expression and equation represented an unknown number. This could be interpreted as understanding that was based on prior arithmetic knowledge as D is often used to denote a missing number in early arithmetic (Herscovics & Linchevski, 1994; Kieran, 1981). As understanding emerged in Grade 9 some said that it was a variable. These results indicate that understanding D as an unknown number appears to be a suitable foundation from which to introduce the concept of any number or variable. We consider this constitutes, in part, pre-algebraic understanding.

Understanding of x in 3x was more cognitively demanding. Herscovics and Linchevski (1994) found that students have difficulty in interpreting concatenated letters. Some students provided explanations that were grounded in their arithmetic knowledge such as stating that x was a multiplication sign. Booth (1988) and MacGregor and Stacey (1993) suggested delaying the omission of the "x" sign,

Data on responses for the commutative and distributive laws from their research showed that the majority of students could not explain commutativity satisfactorily. Inappropriate responses reflected a lack of knowledge of this law. Their results showed that Grade 9, students gave a satisfactory explanation for commutativity. The majority of students in Grades 7 and 8 could not give a satisfactory explanation for the distributive law; most exhibited no knowledge of this law.

For $3(x + 7) = 24$, the majority of students in Grade 7 did not know how to solve the equation. Some of these responses indicated a lack of understanding of brackets; students initially added 3
and 7 to get 10 and subtracted this from 24 to get 14 for x. Arithmetic processes were evidenced by a small number of students who either used trial and error or an inverse method to find what they referred to as "thespace after the x."

Five epistemological obstacles have been observed from the literature, failure to apply the commutative rule properly, where it is applied on division instead of addition, multiplication and subtraction. Failure to use distributive law, cancellation errors especially on rational functions, improper use of and interpretation of equal sign, for example equal sign sometimes mean equivalence but students would evaluate an answer. There is also failure to perform required operation on the algebraic symbols.

2.4. DIDACTICAL OBSTACLES

Students tend to view fractions as isolated digits, treating the numerator and denominator as separate entities that can be operated on independently. The result is an inconsistent knowledge and the adoption of rote algorithms involving these separate digits, usually incorrectly (Behr et al., 1984; Mack, 1990). For example, \( \frac{x+1}{2x^2+6} \) is written as \( \frac{x}{2x^2} + \frac{1}{6} \) which is incorrect.

Didactical obstacles arise from scholastic practices “undermined” by improper habits proposed by teachers to their pupils. In fact, decisions taken by teachers complicate the learning of mathematical concepts: decisions, sometimes derived from the proposals of the textbooks, and are supplied to the pupil, day after day, always and only unambiguous conventional representations which are in this way blindly accepted by the pupil because of the didactic contract established in class (D’Amore, 1999a). In so doing didactical obstacles are faced. In short if the classroom maintains a rigid environment that does not allow the students to establish and discover new
knowledge hence students are easily confused by the teacher’s practices. Didactical obstacles, observed include failure to use the commutative law correctly. Treating numerator and denominator separately.

2.5. Summary

Literature have shown that students’ obstacles in solving partial fractions and other mathematical problems is attributed, to students’ inability to deal with new knowledge they acquire, adding it to the existing one. It has been seen that some knowledge work in other contexts but not in other areas. Also failure to perform arithmetic operations well cause students, face obstacles in problem solving. Didactical obstacles arise due to the instruction being used whilst epistemological obstacles are as a result of new knowledge acquired failing to fit in the already existing one.

CHAPTER THREE

3.0 RESEARCH METHODOLOGY

3.1 Introduction
This chapter presents the research methodology, the sources of data, the study site and population, the sample size and sampling technique, the procedures of data collection, the data gathering tools, the methods of data analysis, ethical considerations and research design that were used in the research.

It is hard to analyse the obstacles faced by students when solving mathematical problems through investigating their written solutions. It may be more productive, when analysing errors, to interview students, noting their verbalisations and thought patterns about the specific problems with which they were faced. In written tests an interview technique may be used to find out the errors which students have made and the obstacles they are facing. A key assumption in this interview technique is that the types of errors students make will be consistent from one session to another. Nevertheless, it seems possible that one-to-one interviews, despite their limitations, do give greater insights into students’ thinking and obstacles which would not be possible purely from an analysis of paper and pencil solutions.

3.2 Research Design.

Research design refers to a plan for selecting objects, research sites and collection procedures to answer the research questions. According to McMillan (1993), research design shows which individual will be studied, when, where and under which conditions they will be studied. The researcher is going to use tests and interviews to collect data from participants.

In this study a case study is to be used. The major goal of this study is to identify the cognitive obstacles and challenges faced when learning partial fractions. According to Jose & Gonzales (1993) case study gives a better and deeper understanding of a phenomenon which helps as a fact-
finding method with adequate and accurate interpretation of the findings. Cohen (1994) stated that case study helps to gather data at a particular point in time with the intention of describing the nature of existing condition or identifying standards against which existing conditions can be compared or determining the relationship that exist between specific events. Therefore a case study helps to identify more into cognitive obstacles and the epistemological, didactical and pedagogical obstacles involved.

3.3 Instruments used to collect data

In this study, tests and interviews were used to collect information on the cognitive obstacles faced by A’ level students when learning partial fractions. The mathematical problems that were chosen have to emphasise various components of the problem-solving process so that each could draw out a variety of problem-solving behaviours causing obstacles in the present research. The problems used in this study can be categorised according to the research literature as “structured problems requiring productive thinking”; that is, tasks where problem-solving techniques must be used by the problem solver. The four problems were assembled from the topic on partial fractions and are going to be used in conjunction with the interviews to probe the students.

A test is an assessment intended to measure a test taker’s knowledge, skill, aptitude or classification in many other topics. It can be delivered orally or on paper. Tests vary in style, rigor and requests. For example, in a closed book test, a test taker is often required to rely on memory to respond to specific items. The researcher is going to give pupils a test to be written on papers by the students.
The researcher prepared a lesson and taught the pupils the topic on partial fractions. After teaching the pupils, the researcher gave the pupils a test and interviews on the topic. The exercise that was given did not to cater for individual differences as all pupils did the task. The above methods were used to elicit data from the population related to the topic under investigation.

### 3.3.1 The Four Problems that were used as part of interview questions

Table 1

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Express ( \frac{4}{(x+2)(x-2)} ) in partial fractions.</td>
</tr>
<tr>
<td>2</td>
<td>Find the values of A, B and C when ( \frac{2+5x+15x^2}{(2-x)(1+2x^2)} ) is expressed in partial fractions.</td>
</tr>
<tr>
<td>3</td>
<td>Put ( \frac{11+7x}{(1-x)(2+x)^2} ) in partial fractions</td>
</tr>
<tr>
<td>4</td>
<td>Evaluate the values of A; B; C; D and E when ( \frac{4x-1}{(x^2+3)^2+(2x+1)} ) is expressed in the form ( \frac{A}{(2x+1)} + \frac{Bx+C}{(x^2+3)} + \frac{Dx+E}{(x^2+3)^2} )</td>
</tr>
</tbody>
</table>

### 3.3.2 Interview Guide

Individual interviews was done with the fifteen students in order to study the development of the conceptions of each student and their thoughts and opinions on solving partial fractions. A sample of fifteen students was selected.
3.3.3 Interview

Interview is a process of communication in which the interviewee gives the needed information orally in a face-to-face with the interviewer. According to Best and Kahn (1993), “the purpose of interviewing people is to find out what is in their mind what they think or how they feel about something”. The students were interviewed why they carry out certain steps and computations and justify their answers.

3.4. Population, sample and sampling techniques to be used in the study

The objective of this study is to gain insights into cognitive obstacles experienced by students when learning partial fractions. A population of twenty five students who are Advanced level student doing mathematics were taught a lesson on partial fractions. A sample of fifteen students wrote a test on partial fractions and was later interviewed. The 15 students were selected using purposive sampling. During the interviews the students were selected randomly. Students were interviewed one by one or individually. The data from the interview was recorded by the teacher. Sampling can be defined as the method or the technique consisting of selection for the study of the so called part, with a view to draw conclusions on the population under study.

3.5 Procedures of collecting data

The format used for the interviews consisted of the following interview guide. On the four questions
3.5.1. Interview guide

Table 1: Interview Guide

<table>
<thead>
<tr>
<th>Item</th>
<th>Responses</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Read the question aloud</td>
<td></td>
</tr>
<tr>
<td>2. What do you need to find? What is the question asking for?</td>
<td></td>
</tr>
<tr>
<td>3. Without doing any working, tell me how are you going to solve this problem? What are you going to have to do in order to solve this problem?</td>
<td></td>
</tr>
<tr>
<td>4. What will you have to use to work out (such and such)? How will you work out to solve the problem? Show me how you solve the problem. Explain to me what you are doing as you solve the problem.</td>
<td></td>
</tr>
<tr>
<td>5. How can you check to see if your answer is sensible? Study the problem again and decide if your answer is sensible.</td>
<td></td>
</tr>
</tbody>
</table>

The interviews were conducted one-to-one by the researcher with the help of another teacher and the responses were recorded. The researcher took approximately half an hour to interview each student. A class instruction was carried out, students and teacher took part in the class instruction. The design included four steps:
3.5.2. Step 1: Class teaching

The students were taught in order to see some obstacles they are facing. The tasks that were chosen for the teaching covered the core parts of the algebra contents on partial fraction decomposition according to the syllabi.

3.5.3. Step 2: The teachers’ analysis of the tests

In collaboration with the other A’ level teachers, based on the test results of the fifteen students selected and analysed. The teacher analysed and describes carefully what they thought and what the students needed to improve and the obstacles they were facing. Teachers also suggested the instruction that was needed to assist the students to develop their knowledge on partial fractions.

3.5.4 Step 3: Interviews

Individual interviews were done in order to study the development of the obstacles and conceptions of each student and their thoughts and opinions after the individual instruction. Here are some examples of the questions the students were asked: How did you think? What does the answer mean? What is the question asking you to do?

The following are groupsof behavioral terms that were used to analyse the interview protocols. These were some of the behaviours that were observed, when solving partial fractions

Structure derived from Newman (1983) and Ransley (1979) as follows
The researcher went through a series of data gathering procedures by using tests and interviews. These procedures helped the researcher to get accurate and relevant data from the population and sample units. Thus, after having letters of authorization from the authorities for ethical clearance, the researcher went to the school for data gathering. Then after making agreement with the concerned participants, the researcher introduced his objectives and purposes. Then, the test and interviews was administered to the students. The participants gave their own answers to each item independently and the data closely assisting and supervising them to solve any confusion regarding the instrument. Finally, the tests and interviewees notes were summarised and made ready for data analysis. While interview was being conducted, to minimize loss of information, the obtained data was carefully recorded and written in a notebook. In addition, the data available in document forms related to tests were collected from the students.
3.6 Data presentation and analysis procedures

Flow chart and tables will be used to analyse the data. The other information was analysed by narration. This chapter reflects on the analysis of the research data, interviews and written tests. The data was collected from the participants as well as associated findings on interviews. The findings are presented on flow chart and tables. First, organizing and noting down of the different categories will be made to assess what types of themes that may come through the instruments to collect data with reference to the research questions. Finally, the findings will be concluded and suggested recommendations will be forwarded. Tables are easy to present information and easy to read.

3.7 Summary

The purpose of this chapter was to show the methodologies that were used in the research to investigate the cognitive obstacles faced by students when learning partial fractions. The chapter also show the population, sample and the instruments that were used, to collect data. The challenges encountered during learning and implementation of concepts when solving the problems. It also look at the research design that was used its advantages and disadvantages, the instruments used to collect data, the population of participants involved.

CHAPTER FOUR

4.0 DATA PRESENTATION, ANALYSIS AND DISCUSSION
4.1. Introduction
The purpose of this research was to investigate the cognitive obstacles faced by students when learning partial fractions and the challenges encountered. Subsequently, this chapter deals with the presentation, analysis and interpretation of data collected on the cognitive obstacles. It contains two major parts; the first part presents characteristics of respondents. The second part deals with the results of findings from the data gathered through the tests and interviews.

4.2. Data presentation

4.2.1. Analysis of Interview Data

The interviews that were conducted with the Advanced level students (related to their obstacles in solving problems). The following categorisations in behavioral terms was used to analyse the interview protocols on how the students solved the problems.

Table 3: Categorisations in behavioral terms used for interview analysis

<table>
<thead>
<tr>
<th>Reading</th>
</tr>
</thead>
<tbody>
<tr>
<td>↓</td>
</tr>
<tr>
<td>Comprehension</td>
</tr>
<tr>
<td>↓</td>
</tr>
<tr>
<td>Strategy know how</td>
</tr>
<tr>
<td>↓</td>
</tr>
<tr>
<td>Transformation</td>
</tr>
<tr>
<td>↓</td>
</tr>
<tr>
<td>Process skill</td>
</tr>
<tr>
<td>↓</td>
</tr>
<tr>
<td>Solution</td>
</tr>
</tbody>
</table>

**Reading**

The students were unable to read the problem and could not distinguish between the words “Evaluate”, Find and Express hence they did not understand the requirements of the question.
Strategy Know-How.

The students were not able to describe a “method” they would use to tackle the problem. Instead the students were using linear form instead of quadratic to decompose partial fractions hence the student were no sure of the method to be used. The students had no idea on how to approach the solution of the problem at all and the type and nature of partial fraction involved.

Transformation

The student were not able to translate the problem into a mathematical form involving partial fraction of linear, quadratic and repeated form. There was an epistemological obstacle of failing to apply the formulae correctly and mixing of concepts, learnt before, such as using linear form of partial fraction where quadratic is needed.

Process Skill

The students were not able to do the mathematics and could not continue with the solution. Students had difficulty with the arithmetic and solving for the constants A, B and C on the problems on linear, quadratic and repeated form.

4.3. RESULTS AND DISCUSSION
Figure 1 below shows a flow chart and accompanying table for the four problems that were used in the interviews. The flow chart provide a summary of the analysis of the interview data. It indicate at which stage the students were unable to proceed with the solution to a problem. The reasons and obstacles found are stated and the frequencies given as percentages. The table gives the breakdown of the occurrences at the various stages for the sample of 15 students interviewed from a population of 25.

The table accompanying the flowchart in Figure 1 shows that the students were interviewed about their attempt to solve partial fraction problems. Of these 15 students, two (i.e., 13%) were classified as “A”. that is, they were unable to proceed beyond the Comprehend stage for the reason that they found the question too confusing with “too many factors and too many unknowns”. 3 that is 20% were unable to proceed beyond the select strategy” stage (B), as they did not know how to proceed with the solution to the problem. Two as a percentage 13% were not unable to formulate the sum of two partial fractions. The obstacles in using process skills were directly related to the problem rather than a general weakness in using this strategy. Of the 3 that is 20%) students, they merely subtracted a smaller number from a bigger number and were unable to translate problem into a mathematical sum. Five which is 33% managed to obtain a solution to the problem. Of these 5 students, three which is 20% obtained incorrect solutions and were classified as...E”; that is, they “formulated the sum of partial fractions incorrectly” The students used inappropriate strategy, merely manipulates numbers.

Table 4:Flow chart showing some of the students’ obstacles
Table 5: Analysis of obstacles from flow-chart

<table>
<thead>
<tr>
<th>N(%)</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
</tr>
</thead>
<tbody>
<tr>
<td>15(100)</td>
<td>2(13)</td>
<td>3(20)</td>
<td>2(13)</td>
<td>3(20)</td>
<td>5(33)</td>
</tr>
</tbody>
</table>
Many epistemological, didactical and pedagogical obstacles, ranging from lack of strategy, transformation were involved. It was observed during the interviews that students were in the habit of trying to solve the current problem using only one strategy for example in expressing \( \frac{4x-1}{(x^2+3)^2(2x+1)} \) in partial fractions. They did not demonstrate any flexibility by trying a strategy and if it did not work, trying another. Students who worked their solutions using an inappropriate strategy shows that they were not aware that the solution was incorrect. Moreover students did not made attempt to find out whether the solutions were correct or whether the solutions satisfied the conditions in the problem of the partial fraction given. Results analysis showed that students were not successful at getting solutions due to the following obstacles:

4.3.1. Lack of comprehension of the question asked

Some students were impeded during their progress when solving the problem since they were notable to comprehend the question; as an illustration, they found the problem confusing with “too many variables and workings” were involved in finding the values of the constants and did not “know how to say” what was involved in a problem. They were unable to visualise and link partial fractions involving the repeated factors, linear and quadratic factors. The students also failed to understand the use of an identity and why it was true for all values of \( x \), whilst an equation is true for only one value of \( x \).

4.3.2. Lack of tactical strategy knowledge

The pupils, were impeded in their progress of solving the problem because they showed having no knowledge of ways in which a partial fraction can be solved when it includes all the types of partial fractions involved all the forms involved.
4.3.3. Inability to translate the problem into a mathematical form involving partial fractions.

Some students, who had a strategy to solve the questions, were impeded in their progress solving the problem by their inability to translate the problem into a mathematical form involving equations, partial fractions and expressing in the required form. For example, for the quadratic problem some students considered it as a linear factor. When solving for the values of A and B, \[ \frac{5x+7}{(x-1)(x+3)} = \frac{A}{x-1} + \frac{B}{x+3}, \] there was an epistemological obstacle of failure to cross-multiply, since students were finding, one common denominator. Students’ arithmetic is poor, they had a difficulty to multiply and perform arithmetic operations.

4.3.4. Failure to apply correct mathematics

Some students, who were able to answer the question, were impeded in their progress in solving the problem by their failure to use the correct mathematics to solve the problem. Other students identified an appropriate operation or sequence of operations but did not know the procedures necessary to carry out these operations accurately. For example, for the linear problem, students were failing to expand \((x - 7)(x + 3)\) in an identity and simplify the answers, resulting in getting incorrect solution. A student merely subtracted a bigger negative number from a small positive number and give the answer as a positive number, for example \(3x - 7x\) is given as \(4x\), which is incorrect. When solving identities students correctly formed two linear, equations but were unable to solve the two equations simultaneously and did not grasp the procedures involved in solving two
linear equations and find the constants. Students also failed to use the distributive property well when solving the problems involving expansion.

The results of the analysis of the data also showed that many times, the solution obtained by the students was incorrect and this could be attributed to the following didactical obstacles.

4.3.5. Inappropriate strategy used on operations

Most students used an inappropriate strategy to solve the questions was number manipulation where students just manipulated the information in the problem by using the four operators (+, -, x, ) to get an answer. Other strategies used in solving the problems were due to approaches used by the teacher in class and have caused students to get wrong answers.

4.3.6. Incorrect formulation of the partial mathematical form

Different students, while solving the problems, formulated the sums incorrectly where they did not include some workings in technical problems.

4.3.7. Computational errors

Computational errors was one of the pedagogical obstacle faced by the students. Several students obtained incorrect solutions due to some careless computations. For example when removing $4x-5$ from $x^2 + 3x - 5$ students incorrectly simplified it as $x^2 + 3x - 5 - 4x - 5$ to get of $x^2 - x - 10$, instead of $x^2+3x - 5 - (4x - 5)$ to get $x^2-x$. Students failed to use the parenthesis around $4x - 5$ when subtracting resulting in wrong polynomials formulated.
Students were not aware that mathematically the parenthesis should be there, but they don’t put and promptly forget that they were there and do the subtraction incorrectly. Improper distribution, the students could not use the distributive property correctly. Two main errors were noted, when students were simplifying the expressions.

The table below show an example of epistemological obstacle that students faced when they were simplifying the results of the partial fraction

**Table 6: Pedagogical obstacles**

<table>
<thead>
<tr>
<th>Example 1: Multiply $4(2x^2 - 10)$</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Correct</td>
<td>Wrong</td>
<td></td>
</tr>
<tr>
<td>$4(2x^2 - 10) = 8x^2 - 40$</td>
<td>$4(2x^2 - 10) = 8x^2 - 10$</td>
<td></td>
</tr>
</tbody>
</table>

The students could not distribute the 4 through the parenthesis. Students often just multiply the first term by the 4 and ignore the second term. This was often done especially when the second term was a number. For some reason, if the second term contains variables students were remembering to do the distribution correctly more often.

Example 2: Multiply $3(2x - 5)^2$

<table>
<thead>
<tr>
<th>Correct</th>
<th>Wrong</th>
</tr>
</thead>
<tbody>
<tr>
<td>$3(2x - 5)^2 = 3(4x^2 - 20x + 25)$</td>
<td>$3(2x - 5)^2 = (6x - 15)^2$</td>
</tr>
<tr>
<td>$= 12x^2 - 60x + 75$</td>
<td>$= 36x^2 - 180x + 225$</td>
</tr>
</tbody>
</table>

Students were not able to identify that in their calculations exponentiation should be performed before you distribute any coefficients through the parenthesis.
The following analysis shows epistemological and didactical obstacles obtained from research.

4.3.7. Imperfect mathematical knowledge

Two students were not successful in obtaining the solution to the problems due to an imperfect knowledge of algebraic and arithmetic manipulations. Students made a lot of additive assumptions, such as \((x + 2)^2 = x^2 + 4\) and \(\frac{1}{x^2+5} = \frac{1}{x^2} \frac{1}{5}\) as a number in partial fractions.

4.3.8. Misinterpretation of the problem

Many students were not able to obtain the solution to the quadratic, linear and repeated problem as they failed to make a link on the concepts, and know the appropriate form required. Therefore, it may be inferred that the cognitive obstacles experienced by Advanced level students who were prevented from getting a correct solution were: (a) inability to comprehend the question posed, (b) lack of strategy knowledge, (c) inability to translate the problem into mathematical form and (d) inability to use the correct mathematics.

Students obtained wrong solutions due to the following reasons: (a) an inappropriate strategy used, (b) incorrect formulation of the mathematical form, (c) computational errors, (d) imperfect mathematical knowledge and (e) misinterpretation of the question.
4.3.9. The table below shows the marks obtained by sample of 15 pupils on the partial fractions problems test given by the teacher.

Table 7: Grouped students’ marks

<table>
<thead>
<tr>
<th></th>
<th>Average</th>
<th>No` of pupils</th>
</tr>
</thead>
<tbody>
<tr>
<td>Low</td>
<td>0 - 20</td>
<td>6</td>
</tr>
<tr>
<td>Average</td>
<td>21 - 40</td>
<td>4</td>
</tr>
<tr>
<td>High medium</td>
<td>41 - 60</td>
<td>3</td>
</tr>
<tr>
<td>Highest scores</td>
<td>61 - 80</td>
<td>2</td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td>15</td>
</tr>
</tbody>
</table>

4.3.10. Table 8: Actual marks of students for test.

<table>
<thead>
<tr>
<th>Code of pupil</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
<th>F</th>
<th>G</th>
<th>H</th>
<th>I</th>
<th>J</th>
<th>K</th>
<th>L</th>
<th>M</th>
<th>N</th>
<th>O</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mark obtained</td>
<td><strong>11</strong></td>
<td><strong>24</strong></td>
<td><strong>39</strong></td>
<td><strong>56</strong></td>
<td><strong>34</strong></td>
<td><strong>16</strong></td>
<td><strong>69</strong></td>
<td><strong>09</strong></td>
<td><strong>58</strong></td>
<td><strong>13</strong></td>
<td><strong>14</strong></td>
<td><strong>17</strong></td>
<td><strong>75</strong></td>
<td><strong>38</strong></td>
<td><strong>58</strong></td>
</tr>
</tbody>
</table>

The data obtained from the test shows that most of the pupils were not able to solve partial fractions problems. Only 33% percent of the students got marks that were above half with the majority getting marks which were below 50%. One of the problems faced by pupils was in ability to solve
algebraic equations, failure to simplify expressions, failure to solve linear algebraic equations, failure to calculate the value of an algebraic expression given certain values for let \( x=3 \) and also failure to manipulate brackets.

Students had a tendency to rush toward a solution before the problem had been clearly defined in the students’ working causing the students to get wrong answers. Categorisation ability to recognise that a problem fits into an identifiable category of problems which run from easily categorisable to uncategorisable forms, especially when students identify the form of partial fraction taken by a certain problem such quadratic, linear and repeated form. For example expressing \( \frac{x+1}{2x^2+6} \) in partial fractions is written as \( \frac{x}{2x^2} + \frac{1}{6} \) by the student which is incorrect. This shows that students relied on his intuition to decompose \( \frac{x+1}{2x^2+6} \) hoping to get \( \frac{x}{2x^2} + \frac{1}{6} \) mentally, his behavior can be described as being passive that is the action was not based on conscious and explicit logical reasoning, justification or sufficient empirical evidence because when he try to simplify it back, a new function is obtained, which is not similar to the original. However the student can only be active if he realise that the assertion is not compatible with the algorithmic knowledge. This indicates that there was a disequilibrium in the students’ mind. Therefore the student can only be mentally active as soon as he started to act on his intuitive knowledge and algorithmic knowledge bases - rechecking his calculations in order to correct the cognitive dissonance. The very act of resolving the perceived cognitive dissonance is considered as a crucial stage in the process of learning mathematics Ausubel (1988).

Some pedagogical obstacles observed on students was, an overdependence on deceptive intuitive mathematical knowledge, focusing on mathematical procedures and processes, of the mathematical ideas and mathematical representations proved to be the causes of stagnation in
knowledge acquisition in the problem solving of the partial fractions. For example. Express \( \frac{4}{(x+2)(x-2)} \) in partial fractions? Students just write, \( \frac{2}{x+2} + \frac{2}{x-2} \). Overreliance on intuitive mathematical knowledge proved to be a cognitive obstacle to the ‘A’ level mathematics students as they solved problems involving partial fractions.

What was of great concern was that although the students had been taught decomposition of functions to partial fractions, intuitions or pre-suppositions remained influential even after instruction.

4.4. Summary

This chapter presents the data that was collected from the research carried out at St Columba’s high. It was intended to establish the obstacles that are faced by students in the learning of partial fractions and their contribution to pupils’ inability to solve partial fractions problems at Advanced Level. The research shows that lack of strategy knowledge, inability to translate the problem into mathematical form, inability to use the correct mathematics. Students obtained incorrect solutions for the following reasons, an inappropriate strategy used, incorrect formulation of the mathematical form, computational errors, imperfect mathematical knowledge, misinterpretation of the problem.
CHAPTER FIVE

5.1. Introduction

This chapter focuses on the findings of the research problems as to the cognitive obstacles faced by students. Further, it discusses the recommendations which could be done. The chapter also gives the conclusion on the findings, summary, references and the appendices that were used in the research.

5.2. Summary of the project including constraints

Knowledge of cognitive obstacles faced by ‘A’ level mathematics students is a key component of mathematics teachers’ knowledge for teaching mathematics referred to as pedagogical content knowledge (Shulman, 1986). This paper reports on cognitive obstacles faced by ‘A’ level mathematics students in understanding partial fractions. Knowledge of students’ cognitive obstacles would enable teachers to provide high-quality instruction for all students.

Cognitive obstacles have been identified in arithmetic and algebra when students learn partial fractions. Others are known to exist for example, the difficulties students experience in solving algebra indicate another major cognitive gap most recently illustrated by MacGregor and Stacey (1993). Other cognitive obstacles experienced by the students was shown by the students’ inability to deal with unknowns when simplifying expressions. It has been seen from the theoretical framework by Vygotsky that different complexes that come when students are learning cause cognitive obstacles to manifest.
There is need for teachers to develop alternative instructional choices that cater for and reduce the obstacles faced by the students. The meaning of the equal sign and identity was interpreted wrongly by the students and this has been confirmed by the results, especially in its use to indicate the idea of calculating the values of constants in a given identity. Teachers’ teaching strategies for the order of operations may cause overgeneralizations by students who may decide that addition takes precedence over subtraction and others thinking that division is commutative.

5.3. Conclusions

When students were asked to justify their procedural approaches the students stated that, the phrases,“express and find” demanded them to embark on just splitting the the rational fraction as done, everyday mathematics. Teachers and mathematics educators should focus on the possible obstacles faced by the students as they interact with the mathematical problem and problem solution. Successful problem solving involves coordinating previous experiences, knowledge, familiar representations and patterns of inference and intuition in an effort to generate new representations and related patterns of inference that resolve the tension or ambiguity that prompted the original problem-solving activity in partial fractions. Teachers must not dismiss students’ solution proposals in class even if it is incorrect but assist the students, to use these experiences to get solutions. Thus, students need to problematize their own learning in order to overcome epistemological and didactical obstacles. Students were also anxious and extremely uncomfortable because they were not able to recall and apply learned concepts in a straightforward way when solving the partial fraction problems.

The results of the study also show that learners had difficulties in translating from one mode of representation of partial fractions to another and this particular phenomenon reveals a cognitive
difficulty that arises from the need to accomplish flexible and competent translation back and forth between different kinds of mathematical representations (Duval, 2002).

It was observed during the interviews that students were in the habit of attempting to solve the current problem using only one method whilst the question needed many strategies. Students were failing to show any flexibility in seeking to solve the problems using more than one method. The obstacles experienced by students have important implications for classroom teachers to remediate the students. This study also shows that students must possess relevant knowledge and be able to coordinate their use of appropriate skills to solve problems. Furthermore, knowledge factors (Kroll & Miller, 1993) such as algorithmic knowledge, linguistic knowledge, conceptual knowledge, and strategic knowledge are vital traits of problem-solving ability. For mathematics teachers to assist students develop problem-solving ability, it is essential that they are aware of their difficulties first and the students must be sensitised of these obstacles.

The results of the study also show that learners had difficulties in translating from one mode of representation of partial fraction form to another and this particular phenomenon reveals a cognitive difficulty that arises from the need to accomplish flexible and competent transformation of partial fractions back and forth between different kinds of mathematical representations (Duval, 2002).

Cognitive obstacles include inability to comprehend the question posed, lack of strategy knowledge, inability to translate the problem into mathematical form and inability to use the correct mathematics.
The constraints of the study included, sample limitation as the students were taken from one school therefore is not able to be generalized to schools or other students from dissimilar settings and that the performance of students in mathematics is also affected by the environments that the students come from. For example some parents have a negative attitude towards mathematics hence they discourage their children that mathematics is a difficult subject.

5.4. Recommendations

According to (Collins 1980), teachers should not accept any mathematical proposition based on intuitive convictions. However they should exploit the learners’ intuitive knowledge which they bring to the learning of mathematics as opportunities for creating cognitive dissonance in the learners’ mind. As soon as learners are helped to resolve these dissonances meaningful learning would take place and obstacles are removed. Teachers need to act as mentors to encourage their students to build thinkable concepts that link together in coherent ways.

‘A’ level mathematics teachers should teach for proceptual understanding. However, if they teach for procedural understanding, there may be long-term drawbacks. The learners may become more procedural and might not develop a process conception. Failure to reify a concept implies that the learners are likely to suffer from cognitive obstacles.

On the basis of the findings obtained and the conclusions drawn, the following recommendations are forwarded to improve obstacles that are faced by the students in their learning of mathematics. Teachers should improve their instructional choices, so that the students understand the concepts learnt, because some obstacles such as didactical are caused by instructional choices of the teacher.
The teachers’ understanding of algebra also influence their ways of teaching partial fractions decomposition and the students’ ways of coping with them. Therefore teachers need to improve their method of instruction to avoid mis-understanding of concepts by students.

Teachers should teach the students to interpret the questions correctly rather than just carry out calculations without understanding the question, in other words rote-learning must be discouraged. Key words such as evaluate, express, find and others should be well explained to the students so that they know the requirements of the questions and what they mean. Failure to comprehend questions cause students to write wrong answers. Teachers should teach using guided discovery, so that students get strategies when solving problems. Teachers should prepare and improve on instructions and manipulation of signs. Students’ methods for solving problems are diverse and can lead to very different patterns of performance than one might expect, so consistent guidance and mentoring must be done by the teachers.


Appendices

Appendix A

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The main purpose of this interview is to gather relevant data to assess the cognitive obstacles faced by students. The response provided are constructive and important for the accomplishment of this
study. So, you are requested to give genuine response. Your response will be used only for academic purpose and remained confidential. The researcher will capture the students’ responses.

<table>
<thead>
<tr>
<th>Item</th>
<th>Responses</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Read the question aloud</td>
<td></td>
</tr>
<tr>
<td>2. What do you need to find? What is the question asking for?</td>
<td></td>
</tr>
<tr>
<td>3. Without doing any working, tell me how are you going to solve this problem? What are you going to have to do in order to solve this problem?</td>
<td></td>
</tr>
<tr>
<td>4. What will you have to use to work out (such and such)? How will you work out to solve the problem? Show me how you solve the problem. Explain to me what are you doing as you solve the problem.</td>
<td></td>
</tr>
<tr>
<td>5. How can you check to see if your answer is sensible? Study the problem again and decide if your answer is sensible.</td>
<td></td>
</tr>
</tbody>
</table>

**Appendix B**

The Four Problems that were used as part of interview questions, to probe the students

Table 2

1. Express \( \frac{4}{(x+2)(x-2)} \) in partial fractions.
2. Find the values of A, B and C when \( \frac{2+5x+15x^2}{(2-x)(1+2x^2)} \) is expressed in partial fractions.

3. Put \( \frac{11+7x}{(1-x)(2+x)^2} \) in partial fractions

4. Evaluate the values of A; B; C; D and E when \( \frac{4x-1}{(x^2+3)^2+(2x+1)} \) is expressed in the form

\[
\frac{A}{(2x + 1)} + \frac{Bx + C}{(x^2 + 3)} + \frac{Dx + E}{(x^2 + 3)^2}
\]

---

**Appendix C**

The following categorisations in behavioral terms was used to analyse the interview protocols on how the students solved the problems (Newman (1983), Table 3.)
Appendix D

The flow chart below shows or is an analysis of challenges that were faced by students.
Comprehend → Find question confusing to many symbols and unknowns (13%)

Select strategy → Lack strategy, do not know how to proceed (20%)

Formulate the partial fraction → Unable to translate problem into partial fractions (20%)

Do the Mathematics → Erroneous calculations, merely, subtract, and add (26%)

Incorrect Formulate the sum incorrectly (33%)

Solution

Table 4

<table>
<thead>
<tr>
<th>N(%)</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
</tr>
</thead>
<tbody>
<tr>
<td>15(100)</td>
<td>2(13)</td>
<td>3(20)</td>
<td>2(13)</td>
<td>3(20)</td>
<td>5(33)</td>
</tr>
</tbody>
</table>

Appendix E

The table below show an example of epistemological obstacle that students faced when they were simplifying the results of the partial fraction
The students could not distribute the 5 through the parenthesis. Students often multiply the first term by the 5 and ignore the second term. This was often done especially when the second term was a number. For some reason, if the second term contains variables students were remembering to do the distribution correctly more often.
**Example 2:** Multiply $3(2x - 5)^2$

<table>
<thead>
<tr>
<th>Correct</th>
<th>Wrong</th>
</tr>
</thead>
<tbody>
<tr>
<td>$3(2x - 5)^2 = 3(4x^2 - 20x + 25)$ $3(2x - 5)^2 = (6x - 15)^2$</td>
<td></td>
</tr>
<tr>
<td>$= 12x^2 - 60x + 75$</td>
<td>$= 36x^2 - 180x + 225$</td>
</tr>
</tbody>
</table>

Students were not able to note that in their calculations exponentiation must be performed before you distribute any coefficients through the parenthesis.
The table below shows the marks obtained by sample of 15 pupils on the partial fractions problems test given by the teacher.

**TABLE 5**

<table>
<thead>
<tr>
<th>Category</th>
<th>Average</th>
<th>No` of pupils</th>
</tr>
</thead>
<tbody>
<tr>
<td>Low</td>
<td>0 - 20</td>
<td>6</td>
</tr>
<tr>
<td>Average</td>
<td>21 - 40</td>
<td>4</td>
</tr>
<tr>
<td>High medium</td>
<td>41 - 60</td>
<td>3</td>
</tr>
<tr>
<td>Highest scores</td>
<td>61 - 80</td>
<td>2</td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td>15</td>
</tr>
</tbody>
</table>

Table of actual marks got by students in a test.

<table>
<thead>
<tr>
<th>Code of pupil</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
<th>F</th>
<th>G</th>
<th>H</th>
<th>I</th>
<th>J</th>
<th>K</th>
<th>L</th>
<th>M</th>
<th>N</th>
<th>O</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mark obtained</td>
<td>11</td>
<td>24</td>
<td>39</td>
<td>56</td>
<td>34</td>
<td>16</td>
<td>69</td>
<td>09</td>
<td>58</td>
<td>13</td>
<td>14</td>
<td>17</td>
<td>75</td>
<td>38</td>
<td>58</td>
</tr>
</tbody>
</table>

**Appendix G**
Test Questions

1. Express \( \frac{2x^2+1}{x(x-1)^2} \) in partial fractions.

2. Evaluate the values of A, B and C by putting \( \frac{3x+8}{(2x+1)(x^2+3)} \) in its partial fraction form.

3. Write \( \frac{x^2+x-4}{(2x+1)(x^2+4)} \) in its partial fractions.

4. Find \( \frac{2}{x^2-1} \) in the form \( \frac{A}{Bx-1} + \frac{C}{Dx+1} \).

5. Express \( \frac{5x-2}{x^2(x+1)} \).

6. Put \( \frac{x^3}{x^2+3} \) in its partial fractions.

Appendix H
APPLICATION LETTER TO SCHOOL

BINDURA UNIVERSITY OF SCIENCE EDUCATION

FACULTY OF EDUCATION

DEPARTMENT OF SCIENCE EDUCATION

TO WHOM IT MAY CONCERN:

PROJECT TITLE: Cognitive obstacles faced by ‘A’ level mathematics students when learning partial fractions: A case study of a school in Manicaland Province

I am Master of Science Education in Mathematics student of the Bindura University of Science Education, who has to carry out a research project which is a partial requirement of the above course.

I am kindly ask your assistance to carry out research at your school. Any assistance offered will be greatly appreciated. I guarantee that no information supplied will be made public without your written permission.

Such research projects are of purely academic purposes only. If required by your institution, a copy of the project will be made available to you, in the hope that the data contained may be of some use to the institution in its future operations.

Thank you for your consideration

Yours sincerely

Trust Jombo