An exploration of students’ understanding of probability using Triad Theory; A Case Study of a school in Chinove Cluster; Mhondoro Ngezi District, Zimbabwe.

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Dedication

To my wife Marvelous Muranganwa and my Daughters Marlyn and Nakiso
Acknowledgement

Preparation of this thesis involved contributions and cooperation of many people. My gratitude goes to my wife, Marvelous, for her moral and economic support. At Bindura University of Science Education I would like to thank my supervisors Mr Chagwiza and Mrs Mutambara whose guidance contributed to the successful production of this research project. Not forgetting Mr Zengeya my project coordinator your efforts are greatly appreciated. My gratitude also goes to Mrs Sora and other School heads in the cluster for assistance and for granting me permission to carry out this research project at Chinove Secondary School and members of staff at the school Mr Makwarimba, Mrs Chitorido, Mr Chigova and Mr Matonga my H.O.D for their assistance in logistical challenges. I would also like to thank all my fellow students for their ideas and contributions towards this research project. Lastly to those who I unintentionally failed to mention by name your assistance in whatever way it came, is greatly appreciated.

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Abstract

This paper reports on how Ordinary Level Mathematics students in Chinove Cluster Mhondoro Ngezi conceptualised and understood the probability topic concepts. This is a qualitative research project that explores the level of probability schema development based on the instrumental and relational understanding. The research utilised questionnaire and interviews as the mostly used instruments. The questionnaires were completed by students selected by solving questions grouped as Intra-Stage, Inter-Stage and Trans-Stage probability questions. The work sheets were used to interview each student addressing question on how, what and why a certain question was answered by the student. The Triad Method was used to analyse the mental constructions made in answering questions and interviews understanding the probability concepts. Some students obtained correct answers to the question in writing but they could not understand how, what and how they did that. Also some used methods that contradict the laws of mathematical arithmetic in division. Results indicated that students could understand Intra-Stage questions better than Trans-Stage questions. Results also indicated that coming out with a correct answer does not necessarily mean a student understood what he or she has written for relational understanding and sustenance. This influences the pedagogy to be used in order to enable the students to understand the concept of the probability methods.
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CHAPTER 1: INTRODUCTION

1.1 Introduction

Understanding of any concept in Mathematics is explained by psychologists Jean Piaget and Lev Vygotsky as the mental constructions of knowledge organized in different forms known as schemas; genetic epistemology. They all agree that students of the same age have different cognitive levels of a particular topic concept. The psychology of schema development study by Jean Piaget and Rolando Garcia in 1983 on the Triad Theory or Dialectical Stage Law is one of the main methods of measuring levels of understanding in many Mathematical concepts such as calculus, probability, geometry and differentiation Wood, Smith and Grossniklaus, (2001). This study is aimed at exploring the Triad Theory’s stages displayed by students in answering probability questions and comparing the mental construction with traditional methods of understanding probability so as to design curriculum and make pedagogical adjustments in the teaching of the topic concepts. This chapter will focus on the background of the problem, statement of the problem, sub-problems, and importance of the study, limitations, delimitations and definitions of terms.

1.2 Background of the study

Probability is a calculation of likelihood of events taking place and has a major bearing in business mathematics and statistics Portman and Richardson, (2006). Probability is applied in the medical odds when estimating the chance of survival or the chance of side effects. In productions of many consumer products, such as, automobiles and electronics, producers utilize reliability theory in the design of the product to reduce the chances of failure. In risk Assessment and Trade on commodity markets, government typically apply probabilistic methods in environmental regulations. This is termed ‘pathway analysis’, that often measures wellbeing using methods that are scholastic in nature and choosing projects to undertake based on statistical analysis of their probable effect on population as a whole. The probability of failure may be closely associated with the product’s warranty.

Probability questions proved to be tough for many candidates at Ordinary Level as indicated by Zimbabwe School Examination Council (ZIMSEC) analysis comments for five consecutive years studied by the researcher. In the June 2009 Mathematics examination
sylabus 4008/4028 for Ordinary Level, the question 11 section B, Mathematics paper 2 examinations of June 2010 question 5a and November 2014 examination had a question number 11.d, combined population, samples, fractions, statistics and probability. The examiners comments reflected that, of the few candidates who attempted these questions, only a few got them correctly. The main weakness identified was that candidates failed to apply the concept of frequency, range and percentages in addressing probability problems and formulating fractions from sample and population and simplifying their answer completely by reducing to the lowest term.

There is even more evidence proving that the topic performance is below expectations. In 2011 the November Mathematics Paper 1 syllabus 4008/4028 question number 25 and November 2010 Mathematics Paper 2 examination analysis on question 9b I quote ‘Candidates were expected to be familiar with a deck of cards and the different types of cards. Many did not do well showing that probability as a topic is not being covered well by leaders” (ZIMSEC, 2010). However, the problem is visible in the candidates’ work hence chances are possible that the leaders are doing their work but candidates have not yet developed enough cognitive schemas to understand and combine concepts, which is a prerequisite for the topic as indicated by most examiners’ comments.

In 2013 November Mathematics Exam Paper 1 syllabus 4028/01 question 22 and Paper 2 Section B question 11b (iv) were on probability. The first question 22a was done very well by most students. In this study, such question is grouped as an Intra-Stage question because it focuses on one variable. However questions 22b and 11b involved the concepts from statistics and inequalities these were not done correctly by most candidates. The examiners pointed out that most candidates failed to distinguish mutually exclusive events from independent events and wrongly selected sample space and applying additional laws (ZIMSEC, 2013).

The topic on probability was a challenge for me when I was a student at Ordinary Level in 2003. Understanding of other topics such as general arithmetic, fractions, sets, statistics and number systems helped me to understand and simplify questions on probability. So best performance on the topic cannot be isolated from essential mental level of skills, that should be developed first before a student can understand better, thus, the researcher will use Triad Methods to explore the levels of understanding of the probability concepts.
Teaching Mathematics has always been my passion and as a teacher of Mathematics the topic of probability takes more time and effort to sink into most students’ mind. The main problem I observe is that the topic involve many concepts from others topics such as General Arithmetic’s of Fractions, Sets, Statistics, and Shapes, to mention a few. If a student has not yet mastered concepts from related topics, solving probability questions might not be easy. The teaching method of the topic varies from student to student basing on prior knowledge in related topic concepts. As a mathematics teacher who strives to be a reflective practitioner, the researcher attempts to determine each learners level and what method works best in his teaching.

The learning of mathematics has proved to be a challenge for most pupils in Zimbabwe as indicates by very low pass rate of about 21% recorded currently in all ZIMSEC registered candidates in 2014 (Herald, 2015). The schemas of many factors integrate to address a problem so there is need to assess student prior knowledge on the major topic at Ordinary level such as Probability. Missing important related schemas retards the understanding of Mathematics concepts. This might be another cause of lower national pass rates.

1.3 Statement of the problem

Students studying mathematics at Ordinary Level are facing difficulties in understanding and solving probability questions and other closely related problems such as statistics due to pedagogy and curriculum which is not influenced by the cognitive schema developmental level of the student on probability topic.

1.4 Research Questions

1. What Triad levels are displayed by students when solving probability questions?
2. How do the students mental constructions compare with the historical understanding of probability concepts?
3. How do students view the Triad Method as significant compared to Traditional methods?
1.5 Aims of Study

The study aimed at bringing knowledge to stakeholders in education on the best methods of categorizing learners using mental cognitive psychogenesis stage according to the Triad Theory as intra, inter and trans. Students exhibiting a similar stage in mental construction will be provided with complementary pedagogical challenges to create disequilibrium and facilitate their learning and curriculum thus facilitates better understanding unique to that category.

As encouraged by the Carnegie Mellon University 2014, ‘we may exhibit an admirable command of content, and possess a dazzling variety of pedagogical skills, but without knowing what’s going on in our students’ heads, that knowledge may be presented and that skill exercised in a vacuum of misunderstanding’ (Carnegie Mellon University, 2014).

1.6 Objective of the study:

The study was focused on achieving the following objectives:

1. to determine the Triad stages displayed by students in answering Probability question at Ordinary level.

2. to compare effectiveness between Triad method and Traditional methods in solving probability problems from a students’ perspective.

1.7 Assumptions

In conducting the study the following assumptions were made

(i) Each student performance was determined by his/her cognitive ability and schema development without influence of other contributions of unfair practice.

(ii) The questionnaire used in the research would be able to extract information sought from the respondents.

(iii) The questionnaire respondents gave truthful and sincere answers to the questions
1.8 **Significance of the study**

This research can provide useful information to researchers in Mathematics Education. It is therefore assumed that the research will be of importance to some stakeholders such as students, teachers, Chinove cluster, Mhondoro Ngezi District, province and at national level. They may benefit in the following ways:

- reducing costs by grouping students exhibiting a similar stage in mental construction and providing them with complementary pedagogical challenges to create disequilibrium and facilitate learning and curriculum. This facilitates better understanding unique to that category therefore resources unique the group can be allocated accordingly without wastage.

- spearheading the child centred approaches that are encouraged by the modern philosophy of education, Secondary School Teachers’ Colleges and action groups such as the Performance Lag Address Programme (PLAP) in Zimbabwe

- clarifying lower pass rate in Ordinary level mathematics than pass rates for other subjects; thus an enquiry into the mental constructions for probability concept finds significance.

The Triad Theory point at a specific Mathematics topic concept which make this method most accurate in identifying the exact level of understanding which calls for curriculum and pedagogy change. The Triad Theory methods assist teachers, administrators and policy makers to channel resources to a more specific cause and make changes for sustenance and equity. There is a link between schemas used in addressing problems on Mathematical concepts. Some topic concepts are not easily understood due to influence of weaker schemas on other related concepts. The Triad Theory will show the cognitive level of each student making it easy to increase students’ performance in Mathematics and, this might increase overall percentage pass rate. This means that teachers may choose media, pedagogies, and curriculum that will be most effective for teaching Mathematics.

1.9 **Limitations of the study**

The following are limitations to the study:

- the study was confined to one school as a case study because of inadequate time financial resources. With adequate resources and time the research can be done on all schools in Zimbabwe. This will increase its applicability and it will be more representative.
-some pupils suspected that they were under study and they behaved differently. This change in behaviour distorts results as pupils were not giving a true reflection of their abilities. The researcher monitored behaviour for each student selected and guided them from disruptive behaviour.

-only ten ZIMSEC registered form four students comprised the research sample. Involving students who were preparing for their final ordinary level examination who had covered most topics will assist to identify areas that need revisiting and assisting pupils.

-data collection in the study was confined to the topic “Probability” found in “O” level mathematics syllabus. This is so because of lack of time which makes it impossible to consider other topics. Various topics were incorporated as fractions, word problems, consumer arithmetic and statistics.

1.10 Delimitations of the study

The study involved ten Ordinary level students at the Chinove Secondary school in the Chinove Cluster; Mhondoro Ngezi District. The school, cluster and district were selected based on the researchers’ convenience so as to reduce travelling and communication cost. The researcher has been teaching Mathematics at the school for four years. The sample size of ten Ordinary Level students was chosen at random, this sample was easy to manage due to the resource limitations for printing at least four questionnaires for each student and time to interview each student in the sample. Those who registered for final form four Ordinary Mathematics examinations were expected to work hard since the topic under study was part of their revision as a result the had opportunity to revise and there are more committed to put more effort thus improving results reliability and applicability for the study. The result will be a true representation of the population. Mathematics is one of the major five Ordinary level subjects in Zimbabwe and it is a prerequisite for entry into university and most formal employment; Mathematics is considered first and above other subjects and it is one of the most failed subjects at O level. The cited reasons by examiners were that students do not meet the expectations due to failure to understand interconnection among many concepts which in this study are termed schemas. So it is worth to find out the actual level of understanding probability. The Triad method will be used to analyze the level. The focus was on one topic probability. Probability is an example of applied mathematical topic that involves and links many topic concepts. So learning probability is worth since other areas
will be revised, assessed and analysed. A simple random selection criterion was used to select the sample from all students who registered for ZIMSEC Ordinary level Mathematics. The sample was small due to lack of funding and time to interview all the students from the cluster. The researcher collected data three times in order to improve result credibility, applicability and reliability.

1.11 Definition of terms:

According to (Aspinwall, Shaw, & Presmeg, 1997) exploration-involves investigation, study, survey, research, search, probe, examination, enquiry, scrutiny and observation of something new. In the study the word refers to the act of searching for the purpose of discovering information or mental resources.

(Tall & Thomas, 2002) Define understanding as Instrumental and relational; instrumental understanding involve choosing and applying rules without knowing why thus it restricts the task to surface representations. Relational understanding -knowing both: what to do and why.

Probability- is a branch of mathematics that studies a numerical measure of the chance or likelihood of an event happening or not happening (Kufakuwonda & Nyamakura, 2010, p. 145). Concept of probability of event (X) mean how one conceives of an object X in a certain perspective, say, by inspection, reflection, analysis and scrutiny.

Dubinsky (1991) Triad-a mechanism which consists in three stages, referred to as Intra, Inter, and Trans, in the development of the connections an individual can make between particular constructs within the schema, as well as the coherence of these connections.

The Intra stage of schema development is characterized by a focus on individual actions, processes, and objects in isolation from other cognitive items of a similar nature. For example, in the probability concept, an individual at the Intra level would tend to focus on a single variable and the various activities that he or she could perform with it.

Inter stage is characterized by the construction of relationships and transformations among these cognitive entities. At this stage, an individual may begin to group items together and even call them by the same name. In the case of probability, the individual might think about adding and multiplying mutually exclusive events and independent event respectively.
Finally, at the Trans stage the individual constructs an implicit or explicit underlying structure through which the relationships developed in the Inter stage are understood and which gives the schema a coherence by which the individual can decide what is in the scope of the schema and what is not. For example, an individual at the Trans stage for the probability concept could construct various connections of independent and mutually exclusive events and statistics.

1.12 Summary

This chapter outlined the problem and how the problem arose. It attempted to divide the problem in researchable sub-problems as well as stating the significance of the study to various stakeholders.
CHAPTER 2: REVIEW OF RELATED LITARETURE

2.1 Introduction

This chapter focuses on literature review whose major purpose is to assist the researcher to define the position the research will occupy within the body of knowledge accumulated to date on the same issue. According to Leedy (1980; 69), literature review is an analysis of materials written by other people that are related to one’s research topic and are useful in providing the researcher with new ideas and approaches which might not have occurred to him or captured by one’s study, and is ideal for the evaluation of one’s research efforts by comparing to related efforts by others.

2.2 The origins of probability

Probability theory was inspired by games of chance and gambling during the 16th century in many places in the world. Dicing, card games and lotteries, private and public, were important social and economic activities (Hald, 2003). Probability theory became popular, and the subject developed rapidly during the 18th century. The major contributors during this period were Christiaan Huygens, Jacob Bernoulli and Abraham deMoivre (Burton, 2007). Currently, in the 21st century, probability theory is used in all kinds of risk assessment in the insurance industry, in medical research, in engineering, in finding the genetic makeup of individuals or populations, in quality control, in investment and every other aspect of human life. Nowadays probability is important as it can be used to determine the expected outcome in any situation, for example, the probability that a person will win the lottery.

2.3 Theoretical framework

Piaget and Garcia’s studies of the schema development in infants called Triad Theory in 1983 paved the way for chances of identifying students’ cognitive levels in mathematical concepts such as probability. The Triad Theory mechanism occurs in three stages explains constructions in the mind, implicating mental representations and transformations in the analysis of schema formations. These stages are: the Intra stage which focuses on a single entity, followed by Inter stage which is ability to identify transformations between objects and Trans stage noted as schema development connecting actions, processes and objects
Wood, et al (2001). The succession intra-, inter-, and trans-, which we find in all domains and at all stages, is the expression of conditions imposed on all cognitive acquisition, namely, the laws of assimilation and equilibration. The Triad methods were applied in many Mathematics topics at Ordinary Level, Advanced Level and University level in many countries as a measure of specific mathematical concepts understanding.

2.4 **Historical understanding misconception**

The historical or traditional understanding of mathematical concept at any level of learning is based on the syllabus and the comments passed by examiners on the analysis of the questions on the probability. They indicate the concept of deep ideas, ordinary learning and ability to recall, apply, research, synthesize and evaluation as measures of levels of understanding. The ordinary learning process has short falls emanating from the reorganization of knowledge which sometimes incorporates mathematically incorrect constructions that are held for a period of time. Generally the ability to visualize in mathematics is thought to be beneficial, but (Aspinwall, Shaw, & Presmeg, 1997) in their journal titled ‘Uncontrollable mental imagery: Graphical connections between a function and its derivative. Educational Studies in Mathematics’, reported on a student whose ability to think about a problem was hampered by an incorrect image. The student had constructed an image of a second-degree polynomial function as having vertical asymptotes. However, he asserted that the domain was all real numbers. This erroneous image caused him to draw a graph of the derivative shaped like a cubic function. This drawing was in conflict with the student’s analytic knowledge that the derivative of a quadratic function should be a line. The images that are referred to by Aspinwall et-al (1997) cognitive which the researcher term the schemas. Also they had established the links to the solving of polynomials. Thus there was a need to study on probability schemas due to establishment of how they inter link with other topics in Applied Mathematics and Statistics. So this study cleared the historical understanding misconceptions arising from incorrect schemas.

Measuring of understanding has many angles such as the methods used by Duffin and Simpson (2000) in a Journal of Mathematical Behaviour titled ‘A search for understanding’ have identified and named three components of understanding as (1) the building, (2) the having, and (3) the enacting. They defined ‘building understanding’ as the formation of connections between internal mental structures. ‘Having understanding’ is said to be the state
of these connections at any particular time and ‘enacting understanding’ as the use of the connections available at a particular moment to solve a problem or construct a response to a question (Duffin & Simpson, 2000). Thus this is the type of understanding that may be visible from students’ work when responding to mathematical tasks. Duffin and Simpson also talked about the breadth and depth of understanding. They described the breadth of understanding to be determined by the number of different possible starting points that the learner may have in solving a problem. The depth may be evidenced by the way the learners could unpack each stage of their solution in more detail by referring to more related concept. In this study understanding is group as instrumental and relational understanding.

2.5 Triad Theory Levels

Piaget and Garcia (1983, 1989) wrote in detail about schemas and their development. In their book Psychogenesis and the History of Science (1983/1989), they discuss the development of a schema progressing through the three stages called the triad. These stages are referred to as the intra-, inter- and trans- stages. The Triad Theory is a measure of cognitive level based on the schema development.

The Triad methods indicate level of a particular student which is defined as what the student know on probability concept. Knowing cognitive level of a particular student can mark the onset of interventions to improve that students’ understanding for further studies and sustenance. The intermediate level of thought is the one termed inter-operational by Piaget. The change from the first stage of thought or intra-operational level to the second level is marked “… by a total reinterpretation of the conceptual foundations” (Piaget & Garcia, Psychogenesis and the History of Science, 1989, p. 109) rather than an increase in the amount of knowledge. The intermediate thinking level is characterised by efforts to find relationships and search for transformations according to various forms of correspondence. According to Piaget and Garcia (1989), such triadic (dialectical) sequences, found in the history of science, occur because there is a need for cognitive equilibrium and each of these three stages represents a particular degree of equilibrium.
According to Kitchener, (1988) in his Journal for General Philosophy of Science, Vol. 19, No. 1 (1988), pp. 157-165 stability of structure involves the notion that actual and potential disturbances are annulled. Thus, it appears that all cognition and this means, in particular, scientific qua epistemic cognition must follow certain laws in its functioning. These laws can be called psychological laws but they are also genetic-epistemological ones, because the human mind works in certain ways that are encapsulated in psychological laws (Kitchener, 1986).

2.5.1 The Intra-Stage Level

At the intra-stage of a schema, particular events or objects are analysed in terms of their properties. Explanations at this level are local and particular. An object in the intra level is not recognized by the learner as necessary, and its form is similar to the form of a simple generalization. The student’s use of, comparison of, and reflection upon isolated ideas leads him or her to the construction of relations and transformation in the inter level (Clark, Cordero, Cottrill, Czarnocha, & DeVries, 1997). In this study the Intra-stage of schema development is characterized by a focus on individual actions, processes, and objects in isolation from other cognitive items of a similar nature. For example, in the probability concept, an individual at the Intra level would tend to focus on a single variable and the various activities that he or she could perform with it.

2.5.2 The Inter-Stage Level

In the inter-stage, the student is aware of the relationships present and can deduce from an initial operation, once he has understood, he can imply other concepts along with it or can coordinate it with similar operations. When a student reflects upon these co-ordinations and relations, new mathematical structures evolve (Garcia, 1983). Through synthesis of the inter-stage level transformations, the student is reconstructing an awareness of the completeness in the schema. The student can construct relationships and transformations among these cognitive entities. At this stage, a student may begin to group items together and even call them by the same name. In the case of probability topic concepts, the student can observe the relationship and transformations in mutually exclusive events and independent event by adding and multiplying them respectively.
2.5.3 The Trans-Stage Level

The Trans stage represents a more complete equilibrium by virtue of being a more stable structure, with the degree of stability depending on the kind and complexity of structure found there. At this level, a student can perceive new global properties that were inaccessible at the other levels, construct an implicit or explicit underlying structure through which the relationships developed in the Inter stage are understood. This gives the schema coherence by which the individual can decide what is in the scope of the schema and what is not. For example, an individual at the Trans stage for the probability concept could construct various connections of independent and mutually exclusive events and statistics.

2.6 The Triad Application

The Triad Theory stages in schema development, is an integral component of APOS theory and has been used in several studies of student understanding in various areas of mathematics. In their study of student understanding of the chain rule on the paper titled ‘Thematization of the calculus graphical schemas’, Clark et al. (1997) found that the APOS theory involving actions, processes, and objects was not adequate for analyzing their data on student understanding but the triad of Piaget and Garcia (1983/1989) was useful in interpreting the levels of understanding. The authors described a student at the Intra-stage as having a collection of rules for differentiation, including some special cases of the chain rule, but not recognizing any relationships among these rules. At the Inter-stage, the student begins to collect these special cases and to realize that they are related, but the student is unaware of the general relationship. Finally, at the Trans-stage, a student has constructed the underlying structure of the chain rule and can determine which instances are not parts of this chain-rule schema.

Dubinsky (1991) believes that an individual at the Trans- stage for the function concept could construct various systems of transformations such as rings of functions, together with the operations included in such mathematical structures. Since the Triad Theory forms a base for APOS method of study it is worth to apply in exploring students’ understanding of probability concept a major tough topic for most students at ordinary level.
On the contrary the understanding of a mathematical concept is explained in this study a constructivist theory of learning in Undergraduate Mathematics Education Research in the Teaching and learning of Mathematics at University level by Dubinsky and McDonald in 2001 with the help and adoption of APOS. The study was a qualitative questionnaire and interview to measure mental constructions. It can be also agreed that mathematical ideas begin with human activity and then proceed to be abstract concepts. It is therefore important for us to understand how the construction of concepts in the mind, lead to abstraction of mathematical knowledge (Dubinsky & McDonald, 2001). This interpretation of the relevant knowledge construction processes is essential since it points to the contributions we get from Triad analysis. These include (1) understanding the importance of human thought, (mental constructions relating to the concept), and (2) pointing to effective pedagogy for a particular concept. This informed the basis of the methodology used in this research.

Dina van Hiele-Geldof and Pierre van Hiele’ tested that idea, and defined and empirically developed the Van Hiele 'levels of abstraction in understanding mathematics', more independent of age, in their two separate theses in 1957 titled ‘Similarities and differences between the theory of learning and teaching of Skemp and the Van Hiele levels of thinking’. The intermediate stage of thought in Geometrical thinking levels of 1957 could be related to part of his original third level (Level 2) or his later second level (van Hiele, 2002). At this stage students are able to not only understand properties of classes and figures but also inter-relate properties of classes and figures. Van Hiele describes his original intermediate level as the level at which definitions are understood whilst proofs of theorems are placed on the subsequent level. Their studies were inspired by Piaget's idea of levels of understanding of mathematics, notably linked to age. Definitions will thus be rooted in the perceptual level and operating with them informally at the conceptual level will lead to formalisation at the abstract level. Both formal definitions and proofs will be considered as being abstract level activities although theorems would generally be higher up the learning spiral than the definitions upon which they are based (Tall & Thomas, 2002).

According to (Colignatus, 2014c) article ‘Confusing math in elementary school’ in Boycottholland.wordpress informed that formal definitions will be considered to form part of the third or abstract level of a spiral. However, at the intermediate level students would still be at the intuitive stage. After topics have been encountered at the perceptual level and the
concept has been firmly established at the conceptual level, formal definitions would become possible at the abstract level. The study informs the researcher on considerations to be addressed using Triad Theory on probability topic so as to have a bearing on the curriculum and pedagogies of student at Ordinary Level mathematics. These considerations will form the yard stick to categorize students into the three triads.

2.7 Triad Methods versus Traditional methods

The case that the response to a question is correct might not mean that all students understood the concept equally as supported by the historical methods of understanding. Students might get similar answers to a probability question using other methods that contradicts the Mathematical laws for example a student can obtain the correct answer for \( \frac{15}{44} \) of \( \frac{1}{4} \) by cancelling the similar numbers \( \frac{15}{44} \) instead of using the common factor factorisation methods and dividing by the highest common factor which is 16. In this case the Triad Methods bring out further cognitive schema analysis of the topic concepts and methods thus a clear relational understanding.

The conception of deep ideas does not require a definite ontological commitment. It is compatible with some forms of Platonism, in particular with “methodological Platonism” (Mac Lane, 1981), with moderate formalism (reduced to formal models) and with our tenet that mathematical knowledge cannot be simply transferred ready-made from the teacher to the learner and has to be actively built by the latter in his/her own mind. We should draw, however, the reader’s attention to the groundlessness of certain inferences. Most of mathematical reasoning is controlled by deep ideas, which prevail over the corresponding formal models in case of a cognitive conflict (Tall & Thomas, 2002). Deep ideas originate from conscious mathematical activities and from reasoning in situations arising in real life, science, and mathematics itself. They form a complex web of concepts linked by a whole host of types of meaning-based relationships which depend on a wide variety of activities of their origins (Zingiswa M. M., 2014). In the process of historical development, after having reached a certain level of maturity, deep ideas keep their identity. In short the idea of deep ideas is measured by Triad Stages in this research since each level of cognitive development cannot only be measured by exam solutions. There are more factors to be considered thus the traditional methods of measuring understanding are insufficient.
Measuring understanding of the concept using exam analysis done in the traditional method leave the idea of the working memory of written work. According to Kellogg’s working memory theory, all the main processes involved in writing depend on the central executive component of working memory. As a consequence, writing quality is likely to suffer if any writing process is made more difficult. As predicted, the quality of the written texts was lower when the text had to be written in capital letters rather than in normal handwriting (Olive & Kellogg, 2002). How can we assess the involvement of the central executive in writing? One way is to measure reaction times to auditory probes presented in isolation (control condition) or while participants are engaged on a writing task. If writing uses much of the available capacity of working memory (especially the central executive) then reaction time should be longer in the writing condition. Olive and Kellogg used this probe technique to work out the involvement of the central executive in the following conditions: (1) Transcription: a prepared text was simply copied, so no planning was required. (2) Composition: a text had to be composed, i.e., the writer had to plan and produce a coherent text. There was a pause in writing when the auditory signal was presented. (3) Composition plus transcription: a text had to be composed, and the participant continued writing when the auditory signal was presented. Olive and Kellogg (2002) found that composition was more demanding than transcription because composition involves planning and sentence generation. The Piagetian Triad is suggested as a mechanism for analysing schema development of a particular mathematical concept and the probability is used as an example. The triad of the intra- inter- and trans- levels of schema development provides the structure for interpreting the students’ understanding of the probability.

2.8 Summary

Most authors who have written cognitive methods of exploring understanding of most mathematical concept seem to express the opinion that Triad Theory provides a much better derivative of instrumental and relational understanding not limited to just knowing the correct stage in the development of concept knowledge. Traditional methods are used to measure level of understanding because there are easy and less time consuming but they are just approximations.
CHAPTER 3: RESEARCH METHODOLOGY

3.1 Introduction

This phase justifies methodological approach, including data collection and analytical techniques; use of qualitative methods: choice of research approach and paradigm: anticipation of ethical issues and how data will be analysed (Kumar, 2005). The nature of the study lies in the ability to explore the students understanding of probability concept using the Triad Theory in the cluster.

3.2 Research Design

The theme of this research is to investigate the Triad Level of Ordinary level students understanding of probability topic concepts. A structure interview and questionnaire is the main used in the descriptive data collection. To obtain the traditional rating level a sample of ten students from Chinove Secondary School is given three tests questionnaire to answer in 30 minutes for each. The tests questionnaires are grouped in three Triads; intra, inter and trans stage. After each tests is marked and recorded each respondent is interviewed based on the student work sheet presentation. The three questions for the interview are how, what and why on all questions set in each triad level. Chikoko & Mhloyi, 1995, p. 69 state that 'questionnaire and interviews used by researchers gather information which they convert into quantifiable data given by respondents. A qualitative research design was used for this study. According to (Cohen, 2007) qualitative research refers to a systematic process of collecting data and logically analysing that data in the form of words. A qualitative research is, therefore, any kind of research that produces findings not arrived at by means of statistical procedures or other means of quantification (Strauss & Corbin, 1990, p. 17). However, the disadvantage is that data collected at Chinove Secondary School cannot necessarily be generalized to the wider population. But accuracy improved when the response options clearly differentiated both types and levels of knowledge (Portman & Richardson, 2006).

3.3 Instruments used to collect data

The researcher used three Triad method instruments for carrying out this research. These are the Probability Triad tests, questionnaire and interviews with a questionnaire. These are the major research tool for this study.
Three tests were prepared each the first was on probability Intra-Stage questions, the second tests was on probability Inter-Stage question and the final tests was on probability Trans-Stage questions. These tests were written at two weeks intervals. The main advantage of tests questionnaire is that they allow students to express their ideas freely on controlled class room situations. Also tests involve reflective thinking on the student’s ability to brain storm on a problem which assists in schema development. The main disadvantage of tests is that they limit students to a particular problem and solution (Chivadze, Wadesango, & Kurebwa, 2012).

The interviews were conducted after the marking to assess pupils understanding of what they were expected on each question. They were asked how, what and why they answered each question in the manner they did. This involves face-to-face question-answer sessions with one student at a time. The interview gives room for further questioning where necessary. The interviews enabled the researcher to observe those non-verbal actions which also reveal a lot about the students’ feeling about the issue under study. Semi structured interviews were used where the researcher was expected to cover every question but had some room to explore participant’s responses by asking for clarification (Leedy, 1980). The advantage is that the researcher has a deeper understanding of the responses and it is good for qualitative data.

A questionnaire was completed by the researcher while conducting an interview. This method does not allow students to deviate in any form or way, they are easy to administer and analyse. The questionnaire has the advantage that it can be administered to large populations thereby enhancing the reliability of the findings. Also the questionnaire provides room for greater anonymity. This ensures that social desirability bias is reduced since there are high chances that the respondents will answer the questions as objectively as possible. A closed ended questionnaire is ideal for calculating statistical data and percentages as the answers set is known (Chivadze, Wadesango, & Kurebwa, 2012).

However a questionnaire has its own weaknesses. Some respondents may fail to understand questions resulting in responses that are out of line with the researcher’s focus of study. Another problem of the questionnaire is that it does not give room for further questioning where it might be necessary.
3.4 Population

According to McMillan (2006) a population is defined as a group of elements or individual objects or events that conform to specific criteria and to which we intend to generalise the results of the research. Chinove cluster form the population from which the sample was selected. There are three secondary schools namely Kaponda Secondary, Chinove Secondary School and Benhura Secondary School. All these schools offer Mathematics at ordinary level and are ZIMSEC centres. The summation of all the students registered for Mathematics in the Cluster is ninety-four (94). With twenty-three (23) students from Chinove Secondary School, thirty-two (32) from Kaponda Secondary School and thirty-nine (39) from Benhura Secondary School. The syllabus used by these schools is the same and they teach calculator version Mathematics syllabus 4028 which includes the topic on probability.

3.5 Sample and Sampling Procedure

To validate this research the researcher selected ten Ordinary Level Mathematics students from all the students studying at Chinove Secondary School in form four who registered for their final ZIMSEC exams in Mathematics. The sample was selected at random from the twenty-three (23) Mathematics candidate. Since probability is being taught at Ordinary level, purposive sampling was used, because it is for a purpose. Simple random sampling, where every member has an equal chance of being selected, was used for students who was given the test and answered the questionnaire. The writer prepared some cards with “yes” or “no” responses and put them in a hat. All the students in the classes were asked to pick a card with his/ her eyes closed. 10 students who picked the “yes” cards became the sample used in the study to attempt answering question in the three triad stages (Zingiswa, 2014).

3.6 Procedures Data Collection

The students selected were given questions in the Intra-stage to answer in the class under examination conditions invigilated by Mr Makwarimba who is a Shona teacher at the school. After the test was marked and recorded each student in the test was interviewed by the researcher using a questionnaire. The same students in the sample were given the other tests in the Inter and Trans stages at two weeks intervals invigilated by Mrs Chitorido and Mr Chigova respectively each tests was followed by interviews after marking. The data collection details proceeded as follows:
3.6.1 Probability Triad Tests Questionnaire.

A test dossier was prepared with questions on the probability topic; the question followed the Bloom taxonomy. In this capacity three groups of questions that fall in the Triads as Intra; Inter and Trans on the topic of probability were answered. The questions in each category were marked and recorded as percentage mark. The mark obtained by each student on each category was presented and analysed as a measure of understanding the probability concept used in the historical method. Tests were used for data collection because tests highlight pupil ability or inability to answer set questions, which could reflect pupils’ understanding of a taught concept.

Questions in the intra-stage were characterized by simple assimilation; they were isolated and separated qualities (e.g., schemes, operations, data and objects) with no relations between these underlying elements, or a coordination of them into a larger unity. Questions in the inter-stage separated monadic elements that are related to each other on the same level with correspondences, co ordinations and transformations established between them. This involves a more complex kind of assimilation - reciprocal assimilation. Finally, questions in the trans-stage are an assimilation involving probability on statistics, in which a hierarchical structure involving internal relations, producing a unity. The test results were analysed and compared with the Triad Methods. This followed the method that was used by (Duffin & Simpson, 2000) study on understanding.

3.6.2 Interviews

This involves face-to-face question-answer sessions with one student at a time. The researcher asked each student to describe how, what and why he/she solved the question and indicates his comment by a tick in the relevant box. Probing might be done on every question. In this study the question…

   How?-refer to the relevance of method in answering a particular question.

   What?-refer to the extent of knowledge and understanding of probability demonstration.

   Why?-refer to the reasons for how and what.

The advantage is that the researcher has a deeper understanding of the topic and good qualitative data collecting skills.
3.6.3 Questionnaire

The researcher used closed ended questionnaires where respondents are restricted to choose among any of the given multiple choice answers, that is excellent, good, fair, bad and worst. A questionnaire was used to ten pupils on their understanding of each Probability Triad Tests question set by describing (How, What and Why) they gave the answer to that question. The researcher provided an option on the questionnaire where the researcher indicated their responses by using a tick in agreement or disagreement, rating each response based on clarity, coherence and confidence of the student. The question (How) refers to the method, (What) refers to the knowledge and (Why) refers to the reason for how and what. This assisted in identifying correct answers obtained due to chance and guessing.

3.7 Data Presentation and Analysis Procedures

A combination of both quantitative and qualitative methods will be used since the closed question questionnaire solicits for quantitative data. The data will be analysed tables constructed and bar graphs will be created for the data described, explained and compared marks from the different Triad stages. Charts inform of bar charts and pie charts will be used to provide a pictorial view of students’ performance. The data from the Probability Triad tests research was analysed using descriptive statistics through the use of frequency tables, pie charts. The majority of students’ overall performance and understanding level on probability topic and bar graph will be shown by the highest number of students in the Triad stage after the interviews and questionnaire results are compared. Each student’s oral exercise through interview will clarify exact Triad level to counter for correct answers that might be obtained by dubious guessing in the written work. The results of the tests will be compared to the interview and questionnaire checking which method is more specific and appropriate in measuring mental constructions which is what students know and can demonstrate. This will indicate the students’ views of the traditional methods of testing understanding of the concepts under study.

3.8 Summary

In a nutshell, the researcher discussed the research methodology he used. The next stage therefore is to describe, discuss, analyse and interpret the data so as to establish findings and address the problem.
CHAPTER 4: DATA PRESENTATION, INTERPRETATION AND ANALYSIS

4.0. Introduction

As a way of presenting data collected during research through the use of questionnaires and interviews, the researcher used tables, pie charts and bar graphs to show responses from the answered probability Triad questions. The data is then analysed and interpreted. The presentation and analysis mainly focused on the assessment of respondents’ instrumental and relational understanding level of probability concepts according to the Triad Stage i.e. Intra-, Inter- and Trans.

4.1.0 Data presentation

4.1.1 Students’ performance in Probability Triad Tests Questions

The table below outlines the pupils’ mark for each Triad-Stage Questions on probability.

Table 4.1 shows that the lowest marks obtained on the intra stage probability question were 30% by student number 9 the highest mark was 100%. At inter stage question the student who score the lowest mark obtained has 20% and the highest mark was 100%. Final the highest and lowest mark for Trans Stage was 100% and 5% respectively.
Table 4.1: Students performance in Probability Triad Questionnaires

<table>
<thead>
<tr>
<th>STUDENT NUMBER</th>
<th>INTRA-STAGE Mark (%)</th>
<th>INTER-STAGE Mark (%)</th>
<th>TRANS-STAGE Mark (%)</th>
<th>AVERAGE MARK %</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>85</td>
<td>65</td>
<td>55</td>
<td>68.33</td>
</tr>
<tr>
<td>2</td>
<td>35</td>
<td>25</td>
<td>15</td>
<td>25.00</td>
</tr>
<tr>
<td>3</td>
<td>70</td>
<td>75</td>
<td>45</td>
<td>63.33</td>
</tr>
<tr>
<td>4</td>
<td>100</td>
<td>90</td>
<td>100</td>
<td>96.67</td>
</tr>
<tr>
<td>5</td>
<td>50</td>
<td>55</td>
<td>35</td>
<td>46.67</td>
</tr>
<tr>
<td>6</td>
<td>100</td>
<td>80</td>
<td>60</td>
<td>80.00</td>
</tr>
<tr>
<td>7</td>
<td>45</td>
<td>40</td>
<td>30</td>
<td>38.33</td>
</tr>
<tr>
<td>8</td>
<td>75</td>
<td>50</td>
<td>50</td>
<td>58.33</td>
</tr>
<tr>
<td>9</td>
<td>30</td>
<td>20</td>
<td>05</td>
<td>18.33</td>
</tr>
<tr>
<td>10</td>
<td>60</td>
<td>40</td>
<td>25</td>
<td>41.67</td>
</tr>
<tr>
<td><strong>AVERAGE</strong></td>
<td><strong>65</strong></td>
<td><strong>54</strong></td>
<td><strong>45</strong></td>
<td><strong>53.66</strong></td>
</tr>
</tbody>
</table>

Figure 4.1 Shows the percentages of marks in each Triad Probability questions.
Each student obtained marks different from the other showing that no students are at the same level of understanding of the probability concepts. This proves that there are different levels of students in the Triads Methods. The marks obtained by most students decreased from intra-stage to trans-stage. This shows that infringement of other concept from missing schema to address the disequilibrium imposed by the additional concept might hinder performance for each student.

4.1.2 Overall tests % pass rates

Figure 4.2 Shows bar chart of pass rate for probability tests

The average mark for the Intra-Stage was 65% and 54% at the Inter-Stage while at the Trans-Stage was 42%. The overall performance in the tests was 53.66%. According to the historical method of measuring understanding level the pass rate of the students is 53.66%. In addition 53.66% of the students have developed mental constructions that enable them to answer any question on the probability concepts. The performance on the Intra-Stage questions which is higher than all other stage provides interpretation of the relevant knowledge construction processes is essential since it points to the contributions we get from Triad analysis (Dubinsky & McDonald, 2001). These include understanding the importance of mental constructions relating to the concept and pointing to effective pedagogy for a particular
concept. Teaching from the simple to complex, known to unknown as students performed better on questions based on local example at this stage.

**4.2. Data analysis on interview and questionnaire**

4.2.1 Probability Triad Intra-Stage questions

The tables below provide an analysis summary on interview and questionnaire filling on oral interview to answer the question how? What? And why he/she solve each probability Intra-stage question.

**Table 4.2: summary analysis on responses made by each student**

<table>
<thead>
<tr>
<th>Question</th>
<th>Excellent</th>
<th>Good</th>
<th>Fair</th>
<th>Bad</th>
<th>Worse</th>
</tr>
</thead>
<tbody>
<tr>
<td>Question 1</td>
<td>How</td>
<td>1</td>
<td>4</td>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>What</td>
<td>2</td>
<td>4</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>Why</td>
<td>0</td>
<td>5</td>
<td>3</td>
<td>1</td>
</tr>
<tr>
<td>Question 2</td>
<td>How</td>
<td>2</td>
<td>4</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>What</td>
<td>1</td>
<td>4</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>Why</td>
<td>0</td>
<td>4</td>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td>Question 3</td>
<td>How</td>
<td>3</td>
<td>3</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>What</td>
<td>3</td>
<td>3</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>Why</td>
<td>1</td>
<td>5</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>Question 4</td>
<td>How</td>
<td>2</td>
<td>4</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>What</td>
<td>1</td>
<td>4</td>
<td>3</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>Why</td>
<td>0</td>
<td>5</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>Question 5</td>
<td>How</td>
<td>4</td>
<td>3</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>What</td>
<td>3</td>
<td>4</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>Why</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>Question 6</td>
<td>How</td>
<td>2</td>
<td>3</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>Question 7</td>
<td>How</td>
<td>2</td>
<td>5</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>Question 7</td>
<td>Why</td>
<td>1</td>
<td>3</td>
<td>3</td>
<td>1</td>
</tr>
<tr>
<td>Question 8</td>
<td>How</td>
<td>2</td>
<td>4</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>Question 8</td>
<td>Why</td>
<td>2</td>
<td>2</td>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td>Question 9</td>
<td>How</td>
<td>3</td>
<td>2</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>Question 9</td>
<td>Why</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td><strong>TOTALS</strong></td>
<td>45</td>
<td>91</td>
<td>61</td>
<td>41</td>
<td>30</td>
</tr>
</tbody>
</table>

Figure 4.3 shows a pie chart of Triad Intra Probability Questions

**Summary of Intra Probability questions**

![Pie chart showing percentages of responses to intra probability questions]
The table above showed the comments shown in answering how, what and why in responses given by students on the Intra-Stage probability questions. Most students in the study had developed a good understanding of the intra-stage questions. The highest numbers of 91 which is 34% of all responses in favour of a good which means most students can simply assimilate, separate and isolate qualities (e.g., schemes, operations, data and objects) with no relations between these underlying elements, or a coordination of them into a larger unity (Dubinsky & McDonald, 2001). The least comments obtained were worst with a total amounting to 30 which were 11% of the entire question how, what and why in Triad probability intra stage. There are other students who are in the intra stage, thus their cognitive schema development need to embrace other related concepts before they moved to the next level the Inter-Stage.

4.2.2 Probability Triad Inter-Stage questions

The tables below provide an analysis summary of all students’ interviews and questionnaires completed to answer the questions of how, what and why he/she solved each probability Inter-stage question.

Table 4.3 summary analysis on responses made by each student

<table>
<thead>
<tr>
<th>Question</th>
<th>Excellent</th>
<th>Good</th>
<th>Fair</th>
<th>Bad</th>
<th>Worse</th>
</tr>
</thead>
<tbody>
<tr>
<td>Question 1</td>
<td>How</td>
<td>0</td>
<td>2</td>
<td>4</td>
<td>3</td>
</tr>
<tr>
<td></td>
<td>What</td>
<td>1</td>
<td>1</td>
<td>4</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>Why</td>
<td>0</td>
<td>1</td>
<td>5</td>
<td>3</td>
</tr>
<tr>
<td>Question 2</td>
<td>How</td>
<td>1</td>
<td>2</td>
<td>4</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>What</td>
<td>1</td>
<td>1</td>
<td>4</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>Why</td>
<td>0</td>
<td>2</td>
<td>4</td>
<td>3</td>
</tr>
<tr>
<td>Question 3</td>
<td>How</td>
<td>1</td>
<td>1</td>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>What</td>
<td>1</td>
<td>1</td>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>Why</td>
<td>1</td>
<td>2</td>
<td>5</td>
<td>1</td>
</tr>
</tbody>
</table>
Figure 4.4 shows a pie chart of Triad Inter Probability Questions

<table>
<thead>
<tr>
<th>Question 4</th>
<th>How</th>
<th>1</th>
<th>1</th>
<th>4</th>
<th>2</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>What</td>
<td>2</td>
<td>0</td>
<td>4</td>
<td>3</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>Why</td>
<td>0</td>
<td>0</td>
<td>5</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>Question 5</td>
<td>How</td>
<td>0</td>
<td>2</td>
<td>3</td>
<td>1</td>
<td>4</td>
</tr>
<tr>
<td></td>
<td>What</td>
<td>1</td>
<td>1</td>
<td>4</td>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td></td>
<td>Why</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>TOTALS</td>
<td>12</td>
<td>19</td>
<td>58</td>
<td>30</td>
<td>31</td>
<td></td>
</tr>
</tbody>
</table>

The table above shows the comments of responses given by students in answering how, what and why on the Inter-Stage probability questions. The least comment was excellent with a total amounting to 12 which is 8% of the entire question how, what and why. This means that 8% of the students can clearly separate monadic elements that are related to each other on the same level with correspondences, co ordinations and transformations established between them. This involves a more complex kind of assimilation - reciprocal assimilation. These students had fully developed schemas to understand probability concepts relationally. The highest numbers of 58 which is 38% of the responses in favour of a fair which means most students probability schemas can fairly address inter stage probability questions.
other students who are in the inter stage, thus their cognitive schemas development need to embrace other related concepts before they move to the final stage. The intuitive stage as suggested by van Hiele (2002) slightly varies with the observed Inter-Stage performance by students on probability questionnaire. These students showed a much higher degree that transcended the intuitive level but they agreed with the perceptual level and the concept has been firmly established at the conceptual level (Colignatus, 2014c).

4.2.3 Probability Triad Trans-Stage questions

The tables below show an analysis summary of all students’ interviews and questionnaire completed to answer the question how what and why he/she solved each probability Trans-stage question.

Table 4.4 summary analysis on responses made by each student

<table>
<thead>
<tr>
<th>Question</th>
<th>Excellent</th>
<th>Good</th>
<th>Fair</th>
<th>Bad</th>
<th>Worse</th>
</tr>
</thead>
<tbody>
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<td>What</td>
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<td>4</td>
</tr>
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<td></td>
<td>Why</td>
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<td>1</td>
<td>1</td>
<td>5</td>
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<td>Question 2</td>
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<td>1</td>
<td>1</td>
<td>2</td>
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<td>3</td>
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<td>3</td>
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<tr>
<td></td>
<td>Why</td>
<td>0</td>
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<td>2</td>
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<td>3</td>
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<td>2</td>
<td>3</td>
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<tr>
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<td>1</td>
<td>3</td>
<td>4</td>
<td>1</td>
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<tr>
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<td>---</td>
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</tr>
<tr>
<td>Why</td>
<td>0</td>
<td>1</td>
<td>2</td>
<td>4</td>
<td>3</td>
</tr>
<tr>
<td>TOTALS</td>
<td>8</td>
<td>16</td>
<td>30</td>
<td>60</td>
<td>36</td>
</tr>
</tbody>
</table>

Figure 4.5 shows a pie chart of Trans Intra Probability Questions

The table above shows the comments of responses given by students in answering how, what and why at the Trans-Stage probability questions. The least comment was excellent with a total amounting to 8 which is 5% of the entire question how, what and why. This means that 5.33% of students can clearly carryout assimilation involving probability on statistics and mensuration in which a hierarchical structure involving internal relations and producing a unity. In contrary to (Mac Lane, 1981) Methodological Platonism that those mathematical ideas cannot be passed on ready made to the student by the teacher. The results indicated that the methods applied on delivery and inquiry of the mathematical concept stems for how the students develop the idea in his/her mind. At this Triad stage, these students had full developed schemas to relationally understand probability concepts. The highest number of 60 which is 40% of the responses in favour of a bad which means most students probability schemas cannot address Tran’s stage probability questions. The response of bad and worst had a total of 96 which is 64% of the question answered. Most students are below the Trans stage, thus their cognitive schema development is below expectations of the Traditional methods supported by the exam analyst. Thus as (Tall & Thomas, 2002) put forward deep
ideas can originate from conscious mathematical activities and from reasoning arising in real life situations. Hence the triad analysis methods clarify relational understanding from performance measures known.

4.2.4 Triad Method overall understanding rates.

Table 4.5 shows the overall results from the Triad Method.

<table>
<thead>
<tr>
<th></th>
<th>Excellent</th>
<th>Good</th>
<th>Fair</th>
<th>Bad</th>
<th>Worst</th>
</tr>
</thead>
<tbody>
<tr>
<td>Intra Stage</td>
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<td>91</td>
<td>61</td>
<td>41</td>
<td>30</td>
</tr>
<tr>
<td>Inter Stage</td>
<td>12</td>
<td>19</td>
<td>58</td>
<td>30</td>
<td>31</td>
</tr>
<tr>
<td>Trans Stage</td>
<td>8</td>
<td>16</td>
<td>30</td>
<td>60</td>
<td>36</td>
</tr>
<tr>
<td><strong>TOTALS</strong></td>
<td><strong>65</strong></td>
<td><strong>126</strong></td>
<td><strong>149</strong></td>
<td><strong>131</strong></td>
<td><strong>97</strong></td>
</tr>
</tbody>
</table>

The results of the interviews and questionnaire responses indicate that students’ understanding of the probability concepts of the intra stage is much better than on the concepts in the Trans Stage. It also indicates that most students fairly understand probability concepts and fewer students excellently understand probability concepts.
Figure 4.6 Shows a Pie Chart for the overall Triads from interview and questionnaire

The pie chart shows each understanding level of probability concept in percentages. The highest percentage of the student level of understanding is fair with 26% and the lowest is excellent with 12%. Therefore students sampled have different understanding levels of the probability concept.

4.2.5 Comparative analysis of the Traditional and Triad method

The probability tests refuted that 53.33% of the students understand probability concepts and passed the tests. While the Triad Methods indicated that students understanding of the concepts varies from excellent to worst. Also 12% and 22% for excellent and good respectively showed that some students understand the concept better than others. These showed mental constructions that they relationally understand probability concepts and can demonstrate employ and connect to other topics in addressing daily activities. Some students obtained correct answers to questions and could not describe how, what and why showed that they cannot understand the probability concept for sustenance. In contrary some students could explain the methods of solving a particular a question correctly in the interview on a question they answered wrongly. This might be attributed to mathematical error in writing which (Aspinwall, Shaw, & Presmeg, 1997) termed the incorrect image or schema hampered
the writing process. However (Olive & Kellogg, 2002) propounded that it is due to the capacity of the working memory a gap between transcription and composition of written work and oral exercise of interviews. To these views traditional methods of measuring understanding is limited. Students viewed the methods as inadequate and do not truly show level of understanding in some students since a correct answer does not clearly show that the student relationally understand what he/she wrote.

4.3. Summary

The data presented from tables and charts clearly shows that each student’s understanding of probability has some levels that differ from Intra to Trans according to the Triad measure of cognitive schema development analysis. A student can be better in the inter questions and worse in the Trans questions an indication of missing schemas for the questions. This is proved so by all data analysis made on the data collected from tests questionnaire and interview questionnaire. Also a correct answer to a question does not necessarily mean that a student relationally understand a mathematical probability concept. Incorrect answers to a given mathematical questions need probing and interviews to be conclusive of what the student knows and understand about any mathematical concept.
CHAPTER 5: SUMMARY, CONCLUSIONS AND RECOMMENDATIONS

5.0. Introduction

This chapter serves to bring into light the summary, conclusion, and recommendations that can be made on the survey that was made on the application of the Triad Theory and Methods in exploring students understanding of probability concept compared to the historical or traditional methods used on the results obtained.

5.1. Summary of the project including constraints

The aim of the study was to explore of students’ understanding of probability using Triad Theory in Chinove Cluster. The study was a case study of three secondary schools base in rural areas of the Mhondoro Ngezi District. The study showed that students understanding of probability vary from Intra, Inter and Tran’s stage based on Triad levels. Also a correct answer or incorrect answer does not conclusively tell whether a student understands or not.

Chapter 1 dealt with the background to the research study, the research, the statement of the problem, research questions, and definition of key terms, limitations and delimitations of the study. Chapter 2 reviewed related literature to the research problem.

Chapter 3, research methodology focused on methods through which data was collected. Sampling procedures, data analysis and management was also carried out. The Probability Triad three tests questionnaires were set, wrote, marked and recorded for the selected students from those registered to sit for mathematics in the ZIMSEC 2015 October/November exams. An interview was conducted guided by the test questionnaire assessing students’ ability to describe and explain the how, what and why questions on every question selected. The researcher completed a closed ended questionnaire.

Chapter 4 presented, analysed and discussed data collected. The tests results, interviews and questionnaires data were presented in the form of bar graphs, pie charts and tables, analysed and compared. The results obtained from the tests given and interviews all agreed on the fact
that mental construction for students vary among students at the same educational level. However the Triad Method was used to analyse the data and it provided a better picture by distributing understanding levels in five categories clearly showing the relational understanding level of probability concept.

Triad of data was presented as Intra, Inter and Trans questionnaire and tests interviews. A student can be better in the intra-stage and inter questions and worse in the Trans questions an indication of missing schemas for the questions. Some students failed to answer questions on the tests questionnaire. The major constrain faced was time to carry out the research interviews varies for each student some were interviewed in the morning, during lunch time and over the weekend.

5.2. Conclusions

According to this study, it can be concluded that relational understanding of probability concepts and other mathematical concepts vary in students at the same educational level. A study of tests results percentages does not represent level of cognitive development of the concepts under study since there are many possibilities that lead to a higher mark such as guessing or the use of methods that violate mathematical laws. Since mathematics is an abstract subject guided by laws; without laws there is no mathematics. Some students could clearly answer the interview the interview questions correctly on a question they wrote a wrong respond. An analysis and measure of mental construction of a certain concept requires a more clear method such as those propounded by the Triad Theory. Although the historical methods only indicate that some students performed better than others in probability tests, this does not mean students that score above others in the tests could equally apply probability concepts in other related real life situation for sustenance. However the historical methods are helpful to create boundaries, reducing sample size and dividing the students. The interviews and questionnaires are more appropriate to measure the correct level of mental constructions. Thus the researcher can conclude that a combination of tests results and interviews will give a better and clear picture of the understanding of a student for sustenance of any concept learnt.
5.3. Recommendations

Following an analysis that was made on collected data to explore the cognitive mental constructions on probability concept, using tests, interviews and questionnaires, it can be said that students construct their own knowledge based on the existing schemas known as mental constructions. Therefore the research can safely make the following recommendations:

- Probability concepts like any other concept require educators to understand each learner’s concept level of prerequisites schemas on topics to achieve relational understanding level. Teachers should not hurry when teaching mathematics concepts but should give time to assess the correct level schema construction of students.

- Employer and recruiting officers of universities are encouraged to consider interviews more than results from ZIMSEC in considering what a potential employee they want since they provided a better and true level of understanding of a particular concept.
References


Answer all questions show all your working clearly on the answer paper

1. A matchbox contains 15 used sticks and 25 unused sticks. What is the probability that a stick if selected is used?

   _____ [1]

2. A trader has 48 mangoes for sale. 18 of them are unripe, if a mango is selected at random from the trader what is the probability of choosing an unripe mango.

   _____ [1]

3. Chinove Secondary School contains 750 boys and 450 girls. A student is chosen at random. What is the probability that a girl is chosen?

   _____ [1]

4. 500 tickets are sold in a state lottery. What is my probability of getting a first prize if I buy 4 tickets?

   _____ [1]

5. A bag contains 2 white balls and 3 red balls. A ball is picked at random what is the probability that it is white?

   _____ [1]
6. A fair six sided die is thrown. What is the probability of getting a 3?

   ____ [1]

7. A card is selected from a pack of 52 playing cards. What is the probability that it is a spared?

   ____ [1]

8. \( F = \{2, 3, 7\} \) and \( T = \{10, 20, 30, 40\} \).
   
   (a) If one element is selected at random from \( F \), write down the probability that it is odd.
   
   (b) If one element is selected at random from \( T \), write down the probability that it is a multiple of 5

   a. ____ [1]
   
   b. ____ [1]

9. In a primary school 70% of the boys and 55% of the girls can ride a bicycle. One child is chosen at random from those who can ride a bicycle. What is the probability that he is a boy?

   ____ [1]

TOTAL 10 MARKS

GOOD LUCK
PROBABILITY TRIAD INTER-STAGE QUESTIONNAIRE

Answer all questions show all your working clearly on the answer paper

1. A card is chosen at random from a pack of playing cards. What is the probability that it is either a heart or the Queen of spades?

\[ \frac{13 + 1}{52} = \frac{14}{52} = \frac{7}{26} \] [2]

2. A die is thrown and a coin is tossed. What is the probability of getting both a six and a tail [2]

\[ \frac{1}{6} \times \frac{1}{2} = \frac{1}{12} \] [2]

3. A letter is chosen at random from the word COMPUTER. What is the probability that it is? Either in the word CUT or in the word ROPE

\[ \frac{2}{11} \] [2]

4. In a primary school 70% of the boys and 55% of the girls can ride a bicycle. If a boy and a girl are chosen at random, what is the probability that both of them can ride a bicycle

\[ 0.70 \times 0.55 = 0.385 \] [2]

5. F = \{2, 3, 7\} and T = \{10, 20, 30, 40\}. If one element is selected at random from F U T, write down the probability that it is either a prime factor of 42 or a multiple of 4

\[ \frac{3}{11} \] [2]
PROBABILITY TRIAD TRANS-STAGE QUESTIONNAIRE

Answer all questions show all your working clearly on the answer paper

1. A fair coin is tossed three times. What is the probability of getting exactly two heads? _____ [2]

2. Tendai and Vimbai take a driving test. The probability that Tendai will pass is $\frac{3}{5}$ and the probability that Vimbai will pass is $\frac{2}{3}$. Calculate the probability that only one of them will pass. _____ [2]

3. The table shows race results in meters of 48 student

<table>
<thead>
<tr>
<th>Distance (in m)</th>
<th>100&lt;x&lt;200</th>
<th>200&lt;x&lt;300</th>
<th>300&lt;x&lt;400</th>
<th>400&lt;x&lt;500</th>
<th>500&lt;x&lt;600</th>
<th>600&lt;x&lt;700</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequency (f)</td>
<td>6</td>
<td>8</td>
<td>10</td>
<td>6</td>
<td>13</td>
<td>5</td>
</tr>
</tbody>
</table>

Two pupils are selected to make a report on the race. Find the probability that both pupils had covered 500m or more in the race. _____ [2]

4. In a primary school 70% of the boys and 55% of the girls can ride a bicycle. If a boy and a girl are chosen at random, what is the probability that one of them can ride a bicycle _____ [2]

5. A gambler tries to throw a stone through the hole. What is the probability of the stone going through?

20 cm

20 cm

7 cm

_____ [2]
INTERVIEW QUESTIONNAIRE

SECTION A

Fill in the relevant information in the space provided

Sex: Male ☐ Female ☐

Age: Years ☐ Mark awarded ☐ %

Triad Stage Intra-Stage ☐ Inter-Stage ☐ Trans-Stage ☐

SECTION B

Indicate your response to each of the given questions by ticking in the appropriate box

KEY: E………. Excellent G………. Good

F………. Fair, B………. Bad W………. Worst

In this study the question

How? - refer to the method relevance in answering a particular question.

What? - refer to the extent of knowledge and understanding of probability.

Why? - refer to the reasons for how and what.

Main Question

The researcher asked each student to describe how, what and why he/she solved the question and indicates his comment by a tick in the relevant box. Probing might be done on every question.

Question 1

<table>
<thead>
<tr>
<th>How?</th>
<th>E</th>
<th>G</th>
<th>F</th>
<th>B</th>
<th>W</th>
</tr>
</thead>
<tbody>
<tr>
<td>What?</td>
<td>E</td>
<td>G</td>
<td>F</td>
<td>B</td>
<td>W</td>
</tr>
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Question 2

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<td>F</td>
<td>B</td>
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<td>E</td>
<td>G</td>
<td>F</td>
<td>B</td>
<td>W</td>
</tr>
<tr>
<td>Question 3</td>
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<td>B</td>
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<td>B</td>
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<td>E</td>
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<td>B</td>
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<tr>
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THANK YOU FOR YOUR TIME.